1. This is not exponential. It is a power function, because the \( t \) is in the base, not the exponent.

2. This is not exponential. It is linear.

3. This is exponential. The initial value is 0.75 and the growth factor is 0.2.

4. This is exponential. Using the rules of exponents, we have

\[
Q = 0.2 \cdot (3^{0.75})^t \\
Q = 0.2 \cdot (3^{0.75})^t \\
Q = 0.2 \cdot (2.280)^t.
\]

The initial value is 0.2 and the growth factor is 2.280.

5. This is already in the form \( Q = ab^t \) with an initial value of \( a = 300 \) and a growth factor of \( b = 3 \).

6. Writing this as

\[
Q = \frac{190}{3^t} \\
= 190 \cdot \left(\frac{1}{3}\right)^t \\
= 190 \left(\frac{1}{3}\right)^t,
\]

so the initial value is \( a = 190 \) and the growth factor is \( b = 1/3 \).

7. Writing this as

\[
Q = 200 \cdot 3^{2t} \\
= 200 \left(3^2\right)^t \\
= 200 \cdot 9^t,
\]

so the initial value is \( a = 200 \) and the growth factor is \( b = 9 \).

8. Writing this as

\[
Q = 50 \cdot 2^{-t} \\
= 50 \left(2^{-1}\right)^t \\
= 50 \left(\frac{1}{2}\right)^t,
\]

so the initial value is \( a = 50 \) and the growth factor is \( b = 1/2 \).

9. This is not exponential growth, but rather linear growth. Every week the price grows by the same, fixed amount of $0.02. It does not grow according to a percentage of its present value.

10. This is exponential growth (or decay) because the amount of drug in the bloodstream goes down by the same factor every four hours.
11. This is exponential growth. Every 3 years, the speed grows to be twice (200%) what it was previously.

12. This is not exponential growth. In fact, the height increases and then decreases, which does not describe exponential behavior. For it to be exponential, the height would need to continue increasing by a constant factor (e.g., doubling every second or something similar).

13. After one year, the population is 
\[ P = MZ. \]

After two years, the population is 
\[ P = MZZ = MZ^2. \]

After three years, the population is 
\[ P = MZZZ = MZ^3. \]

Thus, after \( t \) years, the population is 
\[ P = MZ^t. \]

14. \( B \cdot 1.11^n. \)

15. \( 800 \cdot d^k. \)

16. \( 2000k^5. \)

17. If the investment drops by a third, then two-thirds remain, so \( V_0 \left( \frac{2}{3} \right)^n. \)

18. \( Rs^9. \)

19. \( Nz^9. \)

20. Graph (I) is increasing, so it is not a decaying substance. We know that exponential functions are of the form \( Q = \) (Initial value) \( \cdot (\text{Growth factor})^t. \) Since there is no value of \( t \) that will make \( Q = 0 \) (if \( a, b \neq 0 \), graphs (II) and (IV) also cannot represent exponential decay, because the amounts fall to zero. This leaves graph (III), which does look like exponential decay, decreasing, but not going to zero, and curved.

21. Since we know that an exponential function with a non-zero initial value can never have a value of zero (since any growth factor taken to any power is not zero), only graph (III) might be exponential.

22. A formula for \( V \) in terms of \( t \) is given by \( V = 3000(1.062)^t. \) See Table 10.1.

<table>
<thead>
<tr>
<th>Table 10.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
</tr>
<tr>
<td>( V )</td>
</tr>
</tbody>
</table>

23. (a) At \( t = 0 \), we see that \( Q = 4 \), so either of graphs (III) or (IV) could work, but since \( Q = 4(1.2)^t \) is an increasing function, it must be graph (III).

(b) At \( t = 0 \), we see that \( Q = 4 \), so either of graphs (III) or (IV) could work, but since \( Q = 4(0.7)^t \) is a decreasing function, it must be graph (IV).

(c) At \( t = 0 \), we see that \( Q = 8 \), so either of graphs (I) or (II) could work, but since \( Q = 8(1.2)^t \) increases less quickly than \( Q = 8(1.4)^t \), it must be graph (II).

(d) At \( t = 0 \), we see that \( Q = 8 \), so either of graphs (I) or (II) could work, but since \( Q = 8(1.4)^t \) increases more quickly than \( Q = 8(1.2)^t \), it must be graph (I).
24. 

![Figure 10.1](image)

25. 

![Figure 10.2](image)

PROBLEMS

26. (a) (i) $f(1)$ is the population after one year. The population starts at 2,200,000 and grows by one factor of 1.0211, giving 

$$f(1) = 2,200,000(1.0211) = 2,246,420.$$ 

(ii) $f(2)$ is the population after two years. We have 

$$f(2) = 2,200,000(1.0211)(1.0211) = 2,200,000(1.0211)^2 = 2,293,819.$$ 

(iii) $f(3)$ is the population after three years. We have 

$$f(3) = 2,200,000(1.0211)(1.0211)(1.0211) = 2,200,000(1.0211)^3 = 2,342,219.$$ 

(iv) $f(5)$ is the population after five years. We have 

$$f(5) = 2,200,000(1.0211)^5 = 2,442,103.$$ 

(b) In part (a), the growth factor taken to the power of the year is multiplied by the initial population. For year $t$, we have 

$$f(t) = 2,200,000(1.0211)^t.$$ 

27. (a) (i) Since the population starts at 10,200,000, after one year, we have 

$$f(1) = 10,200,000(0.9995) = 10,194,900.$$ 

(ii) Since the population starts at 10,200,000, after 2 years, we have 

$$f(2) = 10,200,000(0.9995)(0.9995) = 10,200,000(0.9995)^2 = 10,189,803.$$
(iii) Since the population starts at 10,200,000, after 3 years, we have
\[ f(3) = 10,200,000(0.9995)(0.9995)(0.9995) = 10,200,000(0.9995)^3 = 10,184,708. \]

(iv) Since the population starts at 10,200,000, after 5 years, we have
\[ f(5) = 10,200,000(0.9995)^5 = 10,174,525. \]

(b) In part (a), the growth factor taken to the power of the year is multiplied by the initial population. For year \( t \), we have
\[ f(t) = 10,200,000(0.9995)^t. \]

28. (a) \( 10,000(\frac{1}{2})^2 \).
(b) \( 10,000(0.7)^6 \).
(c) \( 10,000(1.5)^t \).

29. The initial value, when \( t = 0 \), is 220,000(1.016)^0 = 220,000. This is the population when the city started. The growth factor is 1.016, which tells us the population increases by a factor of 1.016 every year.

30. (a) After 1 minute, we have
\[ Q = 600(0.962) = 577.2 \text{ grams}. \]

(b) After 2 minutes, we have
\[ Q = 600(0.962)^2 = 555.266 \text{ grams}. \]

(c) There are 60 minutes in one hour, so we have
\[ Q = 600(0.962)^{60} = 58.702 \text{ grams}. \]

31. The value changes by an overall factor of
\[ 1.2(1.3)(0.88) = 1.3728, \]
so it increases by 1.3728 times over the three-year period.

32. We can use a calculator to evaluate \( g(t) \) for \( t = 0, 5, 10, \ldots, 30 \). In year \( t = 0 \) the value is given by
\[ V = g(0) = 3000(1.08)^0 = 3000.00. \]

In year \( t = 5 \) the value is given by
\[ V = g(5) = 3000(1.08)^5 = 4407.98. \]

Similar calculations produce the values in Table 10.2. We plot these points in Figure 10.3. Looking at the table and the graph we see that the investment more than doubles (from $3000 to over $6000) in the first ten years. More precisely, it grows by a factor of 6476.77/3000 = 2.16. In the next ten years, it more than doubles again, from approximately $6500 to almost $14,000, and ten years after that, it more than doubles once again, from about $14,000 to over $30,000. In each case the factor by which it grows is the same: \( 13,982.87/6476.77 = 2.16 \) and \( 30,187.97/13,982.87 = 2.16 \).

| Table 10.2 |
| --- | --- |
| \( t \) (years) | \( V \) (dollars) |
| 0 | 3000.00 |
| 5 | 4407.98 |
| 10 | 6476.77 |
| 15 | 9516.51 |
| 20 | 13,982.87 |
| 25 | 20,545.43 |
| 30 | 30,187.97 |

![Figure 10.3](image-url)
33. The starting value is 400. Since one-fifth of the remaining population leaves each year, four-fifths remain, giving a growth factor of $4/5 = 0.8$. This means $P = 400(0.8)^t$.

34.

$$Q = \frac{50}{2^{\frac{t}{12}}}$$
$$= 50 \left(\frac{1}{2}\right)^{\frac{t}{12}}$$
$$= 50 \left(\frac{1}{2}\right)^{\frac{1}{12}}^t$$
$$= 50(0.9439)^t,$$

so $a = 50, b = 0.9439$.

35.

$$Q = 250 \cdot 5^{-2t-1}$$
$$= 250 \cdot 5^{-2t} \cdot 5^{-1}$$
$$= \frac{1}{5} \cdot 250 \left(\frac{1}{5}\right)^t$$
$$= 50 \left(\frac{1}{25}\right)^t,$$

so $a = 50, b = 1/25$.

36. Writing this as $$Q = \sqrt{300} \cdot 2^t$$ we see that the initial value is $a = \sqrt{300}$ and that the growth factor is $b = 2$.

37. Writing this as $$Q = 40 \cdot 2^{2t} = 40(2^2)^t = 40 \cdot 4^t,$$
we see that the initial value is $a = 40$ and that the growth factor is $b = 4$.

38. Writing this as

$$Q = 45 \cdot 3^{t-2}$$
$$= 45 \cdot 3^t \cdot 3^{-2}$$
$$= 45 \cdot \frac{1}{9} \cdot 3^t$$
$$= 5 \cdot 3^t,$$

so $a = 5, b = 3$.

39. Writing this as

$$Q = \frac{\sqrt{100^t}}{5}$$
$$= \frac{1}{5} \left(100^t\right)^{0.5}$$
$$= 0.2 \left(100^{0.5}\right)^t$$
$$= 0.2 \cdot 10^t,$$

we see that $a = 0.2, b = 10$.

40. (a) Yes, $a = 3$ and $b = 0.5$.

(b) Yes. Writing this as $1 \cdot 8^t$, we have $a = 1$ and $b = 8$. 
(c) Yes. We can rewrite this as
\[ 8^{1-2x} = 8^1 \cdot 8^{-2x} = 8 \left( 8^{-2} \right)^x = 8 \left( \frac{1}{64} \right)^x, \]
so \( a = 8 \) and \( b = 1/64 \).

41. The original equation is in slope-intercept form with \( m = 3 \). We need to write this equation in point-slope form
\[ y = y_0 + m(x - x_0) \]
with \( y_0 = 5 \). We already know that \( m = 3 \). To find \( x_0 \), we solve the original equation to find the value of \( x \) that gives \( y = 5 \):
\[ 5 = 3x + 8 \]
\[ 3x = -3 \]
\[ x = -1, \]
so we have \((x_0, y_0) = (-1, 5)\). This gives
\[ y = 5 + 3(x - (-1)), \]
where \( m = 3 \) and \( x_0 = -1 \).

42. We have
\[ y = 3 \left( x \sqrt[3]{7} \right)^3 \]
\[ = 3 \cdot x^3 \cdot 7\sqrt[3]{7} \]
\[ = 21\sqrt[3]{7}x^3, \]
so \( k = 21\sqrt[3]{7} \), \( p = 3 \).

43. We have
\[ y = 3x^3 - 2x^2 + 4x + 5 \]
\[ = 5 + 4x - 2x^2 + 3x^3 \]
\[ = 5 + x \left( 4 - 2x + 3x^2 \right) \]
\[ = 5 + x \left( 4 + x(-2 + 3x) \right), \]
so \( a = 5, b = 4, c = -2, d = 3 \). Another approach is to multiply out as follows:
\[ y = a + x(b + x(c + dx)) \]
\[ = a + (bx + x^2(c + dx)) \]
\[ = a + bx + cx^2 + dx^3 \]
\[ = a + bx + cx^2 + dx^3. \]

We see from that that \( a, b, c, d \) are simply the coefficients of the terms when placed in order of ascending power.

44. We have
\[ y = \left( \sqrt[3]{3} \right)^t \]
\[ = \frac{1}{5} \left( \left( \sqrt[3]{3} \right)^t \right). \]
10.2 SOLUTIONS

\[ \frac{1}{5} (\sqrt{3})^2 (\sqrt{3})^2 t = \frac{1}{5} (3 \cdot 3) t = \frac{1}{5} 9^t, \]

so \( a = 1/5 \) and \( b = 9 \).

### Solutions for Section 10.2

**EXERCISES**

1. Growth because \( 1.03 > 1 \).
2. Growth because \( 1.372 > 1 \).
3. Decay because \( 0.92 < 1 \).
4. Growth because \( 1.003 > 1 \).
5. Decay because \( 0.81 < 1 \).
6. Decay because \( 0.04 < 1 \).
7. Growth of 8.5% corresponds to a growth rate of 0.085. The growth factor is thus \( 1 + 0.085 = 1.085 \).
8. Growth of 215% corresponds to a growth rate of 2.15. The growth factor is thus \( 1 + 2.15 = 3.15 \).
9. Decay by 46% corresponds to a growth rate of \( -0.46 \). The growth factor is thus \( 1 - 0.46 = 0.54 \).
10. Decay by 99.99% corresponds to a growth rate of \( -0.9999 \). The growth factor is thus \( 1 - 0.9999 = 0.0001 \).
11. The growth factor is equal to the growth rate \( +1 \); so the growth rate = growth factor \( -1 \). Therefore the growth rate is \( 1.7 - 1 = 0.7 = 70\% \).
12. The growth factor is equal to the growth rate \( +1 \); so the growth rate = growth factor \( -1 \). Therefore the growth rate is \( 5 - 1 = 4 = 400\% \).
13. The growth factor is equal to the growth rate \( +1 \); so the growth rate = growth factor \( -1 \). Therefore the growth rate is \( 0.27 - 1 = -0.73 = -73\% \).
14. The growth factor is equal to the growth rate \( +1 \); so the growth rate = growth factor \( -1 \). Therefore the growth rate is \( 0.639 - 1 = -0.361 = -36.1\% \).
15. \( a = 200, b = 1.031, r = 0.031 = 3.1\% \).
16. \( a = 700, b = 0.988, r = b - 1 = -0.012 = -1.2\% \).
17. \( a = \sqrt{3}, b = \sqrt{2}, r = b - 1 = \sqrt{2} - 1 \approx 0.4142 = 41.42\% \).
18. \( a = 50, b = 3/4, r = b - 1 = -1/4 = -0.25 = -25\% \).
19. \( a = 5, b = 2, r = b - 1 = 1 = 100\% \).
20. This investment is initially worth \$800 and it grows by 7.3\% per year.
21. This investment is initially worth \$2200 and grows by 21.1\% per year.
22. Note that this function is linear, not exponential. The initial value is \$4000, and it grows by \$100 per year.
23. The initial value is \$8800, and it decreases by 4.6\% per year.
24. The population is \( 2(1.03)^t \) million.
25. The value of the investment is \( 5(1.3)^t \) million dollars.
26. The initial quantity is not given; suppose it is \( Q_0 \). Then

\[
\text{Quantity remaining} = Q_0 (1 - 0.02)^5 = Q_0 (0.98)^5.
\]
27. The rate at which the cost is increasing is not given; suppose it is $r$. Then

\[ \text{Cost} = 2(1 + r)^{10} \text{ million dollars}. \]

28. This expression represents a quantity of dollars growing by six percent for each unit change in $t$. It could be a bank account with 6% interest.

29. This expression represents a quantity of matter shrinking by three percent for each unit change in $x$. It could be a radioactive substance decaying at 3% per day.

30. This expression represents a population of people growing by 0.6 percent for each unit change in $y$. It could represent a population growing by 0.6% per year.

31. This expression represents a quantity of money shrinking by two percent for each unit change in $a$. It could represent the value of a piece of equipment that depreciates at 2% per year.

32. A formula for $V$ in terms of $t$ is given by $V = 3000(1.062)^t$. See Table 10.3.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$ ($)</td>
<td>3000</td>
<td>3186.00</td>
<td>3593.31</td>
<td>4052.69</td>
<td>5474.78</td>
<td>13,496.90</td>
<td>60,722.14</td>
</tr>
</tbody>
</table>

33. If it decreases by 8.3% = 0.083, then $1 - 0.083 = 0.917$ remains. Thus, a formula for $V$ in terms of $t$ is given by $V = 200(0.917)^t$. See Table 10.4.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$, $$1000$s</td>
<td>200.0</td>
<td>154.2</td>
<td>129.7</td>
<td>84.1</td>
<td>22.9</td>
<td>2.6</td>
</tr>
</tbody>
</table>

34. We find the balance in years $t = 0, 1, 2, 3$. Since the account starts with $5000, the balance in year 0 is $5000. At $t = 1$, we have

Balance $= 5000 + 12\% (5000) = 5600.00$.

At $t = 2$, we have

Balance $= 5600 + 12\% (5600) = 6272.00$.

At $t = 3$, we have

Balance $= 6272 + 12\% (6272) = 7024.64$.

![Figure 10.4]
35. If the population doubles during the first year, it must grow by a factor of at least 2. Thus, (c) and (d) are possible: (c) gives a growth factor of $1 + 100\% = 2$ and (d) gives a growth factor of $1 + 200\% = 3$.

36. Writing this as

\[ Q = 90 \cdot 10^{-t} = 90 \left(10^{-1}\right)^t = 90 \cdot 0.1^t, \]

we have $a = 90, b = 0.1, r = b - 1 = -0.9 = -90\%$.

37. $a = 1/40 = 0.025, b = 10, r = b - 1 = 9 = 900\%$.

38. 

\[ Q = \frac{210}{3 \cdot 2^t} \]

\[ = \frac{210}{3 \cdot \frac{1}{2^t}} \]

\[ = 70 \left(\frac{1}{2}\right)^t, \]

so $a = 70, b = 1/2, r = b - 1 = -0.5 = -50\%$.

39. 

\[ Q = 50 \left(\frac{1}{2}\right)^{t/25} \]

\[ = 50 \left(\frac{1}{2}\right)^{\frac{1/25}{t}} \]

\[ = 50(0.9727)^t \hspace{1cm} \text{using a calculator}, \]

so $a = 50, b = 0.9727, r = b - 1 = -0.0273 = -2.73\%$.

40. 

\[ Q = 2000 \left(1 + \frac{0.06}{12}\right)^{12t} \]

\[ = 2000 \left(1.00512\right)^t \]

\[ = 2000(1.0617)^t \hspace{1cm} \text{using a calculator}, \]

so $a = 2000, b = 1.0617, r = b - 1 = 0.0617 = 6.17\%$.

41. The population decreases each minute to $3/4$, or 75\%, of what it was before. Since 25\% of the population is removed each minute, we say it decreases by 25\%.

42. (a) The population was 222,000 in 2000; it was growing at 5.6\% per year.

(b) In 2010, when $t = 10$, we have

\[ \text{Population} = 222(1.056)^{10} = 382.818 \text{ thousand}. \]

Thus, the population in 2010 is predicted to be 382,818.

43. (a) The tablet contains 50 mg. Quinine in the body decays at a rate of $1 - 0.23 = 0.77 = 77\%$ per day.

(b) After 12 hours (1/2 day), we have

\[ \text{Quinine} = 50(0.23)^{1/2} = 23.979 \text{ mg}. \]

44. In each year, the population is found by adding 2.53\% to the previous year’s population.

(a) The population in 2006 is

\[ 13,900,000 + 2.53\%(13,900,000) = 14,251,670. \]

(b) The population in 2007 is

\[ 14,251,670 + 2.53\%(14,251,670) = 14,612,237. \]
(c) The population in 2008 is
\[14,612,237 + 2.53\%(14,612,237) = 14,981,927.\]

(d) The population in 2009 is
\[14,981,927 + 2.53\%(14,981,927) = 15,360,970.\]

The population in 2010 is
\[15,360,970 + 2.53\%(15,360,970) = 15,749,603.\]

Note, however, that if we don’t round off to the nearest person each time, as we have done, that the population in 2010 comes to 15,749,602.

45. After 1 year, we have
\[2000 - 5.47\%(2000) = 2000 - 109.4 = 1890.6 \text{ mg}.\]

After 2 years, we have
\[1890.6 - 5.47\%(1890.6) = 1890.6 - 103.4 = 1787.2 \text{ mg}.\]

46. This quantity is the factor by which the population of Austin has increased in 3 years.

47. This quantity is the factor by which the population of Bismark shrinks each year.

48. Since \((1.03)^2\) represents growth factor of the population of Phoenix over 2 years, the quantity \((1.03)^2 - 1\) represents the fraction by which the city has grown in two years.

49. Yes. For every 1-unit increase in \(x\), the value of \(y\) is 7 times greater. We have \(y = 1 \cdot 7^x\).

50. No. If we have \(y = ab^x\), the value of \(y\) can be 0 only when \(a\) or \(b\) is 0, because the product of two numbers is 0 only if one of them is 0, and \(b^x\) cannot be 0 unless \(b\) is 0. However, if either \(a\) or \(b\) is 0, then \(y\) is always 0, so the other values in the table are not consistent with the first pair in an equation of the form \(y = ab^x\).

51. Yes. For every 5-unit increase in \(x\), we see that \(y\) is 0.9 times its previous value. For the 10-unit increase in \(x\), we see that \(y\) is 0.81 = \((0.9)^2\) times its previous value. We have \(y = 5(0.9)^{\frac{x-4}{5}} = 5.440(0.979)^x\).

52. The investment’s value changes by a combined factor of
\[(1.129)(1.092)(1.113)(0.955)(0.864) = 1.1322\]

over the five-year period, so its value increases by 13.22%.

---

**Solutions for Section 10.3**

**EXERCISES**

1. (a) The average rainfall in each month is in Table 10.5.

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rainfall (in)</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>

(b) In month \(n\),
\[\text{Average rainfall} = 2^{n-2} \text{ inches for } 2 \leq n \leq 6.\]

(c) Total average rainfall = 1 + 1 + 2 + 4 + 8 + 16 = 32 inches.
2. After a five years, the investment increases by a factor of 1.12, and so

\[
\begin{align*}
b^5 &= 1.12 \\
b &= 1.12^{1/5} \\
&= 1.02292.
\end{align*}
\]

The starting value is \(a = 12,000\) so \(V = 12,000(1.02292)^t\).

3. We have

\[
500 \cdot 2^{t/7} = 500 \left(2^{1/7}\right)^t
\]

so \(a = 500, b = 2^{1/7} = 1.1041, r = b - 1 = 0.1041 = 10.41\%\).

4. Assume that the same initial amount of \(S_0\) is deposited in each account. At the end of \(t\) months, the first account has a balance \(a(1.02)^t\) and the second account has a balance \(ab^t\). After 3 months, the second account has a balance of \((1.07)a\). So we get \((1.07)a = ab^t\). This gives \(b^3 = 1.07\). Taking the cube root of each side, we get \(b = 1.023\). Since \(1.023\) is bigger than \(1.02\), the second account is better.

5. Let the initial height of the sunflowers be \(a\). The first sunflower grows at a rate of \(1\%\) per day; its height can thus be given by \(m = a(1.01)^d\), where \(d\) is time in days. The second flower grows at a rate of \(7\%\) per week; its height can be given by \(n = a(1.01)^w\), where \(w\) is time in weeks. Since 1 week = 7 days, \(d = 7w\). We can now rewrite the first formula as \(m = a(1.01)^7w = a(1.01^7)^w = a(1.072)^w\).

Since the growth factor of the first plant is greater than the growth factor of the second, the first flower is growing faster.

At the end of 1 week, the first plant will have a height of \(m = a(1.072)^1 = 1.072a\), whereas the second plant has a height of \(1.07a\). These heights are very close to being the same. At the end of 5 weeks, the height of the first plant will be \(a(1.072)^5 = 1.42a\), whereas the height of the second flower is \(a(1.07)^5 = 1.4a\). Again, these heights are close, with the first sunflower being just a little taller than the second.

6.

\[
Q = \frac{1}{3} \cdot 2^{1/3}
\]

so \(a = 1/3, b = 2^{1/3} = \sqrt[3]{2}\).

7.

\[
Q = -\frac{5}{3^t}
\]

so \(a = -5, b = 1/3\).

8.

\[
Q = 7 \cdot 2^t \cdot 4^t
\]

so \(a = 7, b = 8\).
Chapter Ten /SOLUTIONS

PROBLEMS

25. (a) The half-life is 16. The population starts at 15. The number starts at 17. The value starts at 13. The population starts at 12. 11.

We see that the population initially numbers 24. (a) Using the exponent laws, we have

\[ Q = 4 \left(2 \cdot 3^t\right)^3 \]

\[ = 4 \cdot 2^3 \left(3^t\right)^3 \]

\[ = 32 \cdot 3^{3t} \]

\[ = 32 \cdot 27^t, \]

so \( a = 32, b = 27. \)

10. We see that the population initially numbers \( a = 1200 \) and that the growth rate is \( r = b - 1 = 0.985 - 1 = -1.5\%. \)

11. The investment’s value is initially 3500 and it doubles in size every 7 years

12. The population starts at \( a = 1000 \) and increases by a factor of \( b = 2 \) every \( T = 12 \) years, if \( t \) is in years.

13. The value starts at \( a = 400 \) and increases by a factor of \( b = 3 \) every \( T = 4 \) years, if \( t \) is in years.

14. The number starts at \( a = 50 \) and increases by a factor of \( b = 5 \) every \( T = 15 \) years, if \( t \) is in years.

15. The number starts at \( a = 6000 \) and is multiplied by a factor of \( b = 2/3 \) every \( T = 2 \) months, if \( t \) is in months.

16. The population starts at \( a = 250 \) and changes by a factor of 1/2 every \( T = 6 \) years, if \( t \) is in years. This gives \( Q = 250(1/2)^{t/6} = 250 \cdot 2^{-t/6}, \) so \( b = 2. \)

17. The value starts at \( a = 120,000 \) and changes by a factor of 0.75 every \( T = 2 \) years, if \( t \) is in years. This gives \( Q = 120,000(0.75)^{t/2} = 120,000(4/3)^{-t/2}, \) so \( b = 4/3. \)

18. The amount starts at \( a = 200 \) and changes by a factor of 4/5 every \( T = 90 \) minutes, if \( t \) is in minutes. This gives \( Q = 2200(4/5)^{t/90} = 200(5/4)^{-t/90}, \) so \( b = 5/4. \)

19. The number starts at \( a = N \) people and changes by a factor of 0.9 every \( T = 5 \) days, if \( t \) is in days. This gives \( Q = N(0.9)^{t/5} = N(10/9)^{-t/5}, \) so \( b = 10/9. \)

20. The value in year \( t_0 = 1995 \) is \( a = 800 \) and it grows by a factor of \( b = 1.021 \) each year.

21. The number is \( a = 5 \) million at time \( t_0 = 3 \) and it grows by a factor of \( b = 2 \) every hour.

22. The value in year \( t_0 = 7 \) is \( a = \$200,000 \) and it grows by a factor of \( b = 1.082 \) each year.

23. The amount at time \( t_0 = 8 \) is \( a = 200 \) and its growth factor is \( b = 0.825. \)

PROBLEMS

24. (a) Using the exponent laws, we have

\[ (4)^{t/12} = (2^2)^{t/12} = 2^{2 \cdot t/12} = 2^{t/6}. \]

\[ (16)^{t/24} = (2^4)^{t/24} = 2^{4 \cdot t/24} = 2^{t/6}. \]

Thus,

\[ 15(2)^{t/6} = 15(4)^{t/12} = 15(16)^{t/24}. \]

(b) When \( t = 6, \) the population is doubled, since

\[ P = 15(2)^{6/6} = 15 \cdot 2 = 30. \]

Thus, formula I shows that the population doubles in 6 years.

(c) Formula II shows that the population quadruples (multiplied by 4) in 12 years.

Formula III shows that the population is multiplied by a factor of 16 in 24 years.

25. (a) The half-life is 1 week.

(b) In 10 days, the quantity has decayed to 1/4 of the original quantity. Since \( 1/4 = (1/2)^2, \) the quantity has decayed by 1/2 twice. Thus, the half-life is 10/2 = 5 days.

(c) The quantity has decayed to 1/8 of the original in 6 weeks. Since \( 1/8 = (1/2)^3, \) the quantity has decayed by 1/2 three times. Thus, the half-life is 6/3 = 2 weeks.
26. The population takes 11 years to double. In the first 11 years, it doubles. In the next 11 years, the population doubles again, so it quadruples in 22 years.

27. The compound decays to $25\% = \frac{1}{4}$ of its original quantity in 90 minutes. Thus, the half-life is 45 minutes.

Thinking of the 90 minutes as two 45-minute periods in sequence, the compound is reduced by $\frac{1}{2}$ in the first 45 minutes and reduced by $\frac{1}{2}$ again in the second 45 minutes. Thus, in 90 minutes, the compound is reduced by $\frac{1}{4}$.

28. An object which decays more slowly has a longer half-life. Since

- $20(0.92)^t$ decays 8% per unit of time
- $120(0.98)^t$ decays 2% per unit of time
- $0.27(0.9)^t$ decays 10% per unit of time
- $90(0.09)^t$ decays 91% per unit of time,

we have the order of increasing half-lives in Table 10.6.

**Table 10.6**

<table>
<thead>
<tr>
<th>Smallest</th>
<th>Largest</th>
</tr>
</thead>
<tbody>
<tr>
<td>$90(0.09)^t$</td>
<td>$120(0.98)^t$</td>
</tr>
<tr>
<td>$0.27(0.9)^t$</td>
<td>$20(0.92)^t$</td>
</tr>
</tbody>
</table>

29. Population $q$ grows by 3% per hour, compounded hourly, so each day it grows by a factor of $1.03^{24} = 2.033$. Thus, it more than doubles every day, so it grows faster than population $m$. For any initial populations, in the long term, $q$ has the larger population.

30. Substance $A$ has a half-life of 17 years, so the quantity of $A$ decreases to 50% in 17 years. Since substance $B$ decreases at a rate of 30% per decade, the quantity of $B$ is multiplied by a factor of 0.7 each decade. In 17 years = 1.7 decades, the quantity of substance $B$ is multiplied by $0.7^{1.7} = 0.54 = 54\%$. Thus, substance $B$ decays more slowly than substance $A$, so, in the long term, there is less of $A$.

31. (a) Here, $a = 400$, $b = 2$, $T = 4$. This tells us that the population begins at 400 and that after 4 years it grows by a factor of 2. To see this, let $t = 4$:

$$P = 400 \cdot 2^{4/4} = 400 \cdot 2^1 = 800.$$  

We conclude that the population doubles every 4 years.

(b) We have

$$P = 400 \cdot 2^{t/4} = 400 \cdot \left(2^{1/4}\right)^t = 400 \cdot 1.1892^t,$$

so the annual growth rate is 18.92%.

32. (a) Here, $a = 800$, $b = 2$, $T = 15$. This tells us that the population begins at 800 and that after 15 years it grows by a factor of 2. To see this, let $t = 15$:

$$P = 800 \cdot 2^{15/15} = 800 \cdot 2^1 = 1600.$$  

We conclude that the population doubles every 15 years.

(b) We have

$$P = 800 \cdot 2^{t/15} = 800 \cdot \left(2^{1/15}\right)^t = 800 \cdot 1.0473^t,$$

so the annual growth rate is 4.73%.
33. (a) Here, \( a = 80, b = 3, T = 5 \). This tells us that the population begins at 80 and that after 5 years it grows by a factor of 3. To see this, let \( t = 5 \):
\[
P = 80 \cdot 3^{5/5} = 80 \cdot 3^1 = 240.
\]
Likewise, letting \( t = 5 \) then \( t = 10 \) gives
\[
P = 80 \cdot 3^{10/5} = 80 \cdot 3^2 = 720
\]
\[
P = 80 \cdot 3^{15/5} = 80 \cdot 3^3 = 2160.
\]
We conclude that the population triples every 5 years.

(b)
\[
P = 80 \cdot 3^{t/5}
\]
\[
= 80 \left(3^{1/5}\right)^t
\]
\[
= 80 \cdot 1.2457^t,
\]
so the annual growth rate is 24.57%.

34. (a) Here, \( a = 75, b = 10, T = 30 \). This tells us that the population begins at 75 and that after 30 years it grows by a factor of 10. To see this, let \( t = 30 \):
\[
P = 75 \cdot 10^{30/30} = 75 \cdot 10^1 = 750.
\]
We conclude that the population grows tenfold every 30 years.

(b)
\[
P = 75 \cdot 10^{t/30}
\]
\[
= 75 \left(10^{1/30}\right)^t
\]
\[
= 75 \cdot 1.0798^t,
\]
so the annual growth rate is 7.98%.

35. (a) Here, \( a = 50, b = 1/2, T = 6 \). This tells us that the population begins at 50 and that after 6 years it is reduced by a factor of 1/2. To see this, let \( t = 6 \):
\[
P = 50 \left(\frac{1}{2}\right)^{6/6} = 50 \left(\frac{1}{2}\right)^1 = 25.
\]
We conclude that the population is halved every 6 years.

(b)
\[
P = 50 \left(\frac{1}{2}\right)^{t/6}
\]
\[
= 50 \left(\left(\frac{1}{2}\right)^{1/6}\right)^t
\]
\[
= 50 \cdot 0.8909^t,
\]
so the annual growth rate is \( r = -0.1091 = -10.91\% \).

36. (a) Here, \( a = 400, b = 2/3, T = 14 \). This tells us that the population begins at 400 and that after 14 years it decreases by a factor of 2/3, which means it drops in size by 1/3. To see this, let \( t = 14 \):
\[
P = 400 \left(\frac{2}{3}\right)^{14/14} = 400 \left(\frac{2}{3}\right)^1 = 266.67.
\]
We conclude that the population decreases in size by 1/3 every 14 years.
(b)

\[ P = 400 \left( \frac{2}{3} \right)^{t/14} \]
\[ = 400 \left( \left( \frac{2}{3} \right)^{1/14} \right)^t \]
\[ = 400 \cdot 0.9715, \]

so the annual growth rate is \( r = -0.0285 = -2.85\% \).

37. The starting value is \( a = 2000 \). The growth factor is \( b = 1.0058 \). The growth rate is \( r = b - 1 = 0.0058 = 0.58\% \).

38. We have

\[ V = 500 \left( 3 \cdot 2^{t/5} \right) \]
\[ = 1500 \left( 2^{1/5} \right)^t \]
\[ = 1500 \cdot 1.1487^t, \]

so \( a = 1500, b = 1.1487, r = b - 1 = 0.1487 = 14.87\% \).

39. We have

\[ V = 5000(0.95)^{t/4} \]
\[ = 5000 \left( 0.95^{1/4} \right)^t \]
\[ = 5000(0.9873)^t, \]

so \( a = 5000, b = 0.9873, r = b - 1 = -0.0127 = -1.27\% \).

40. We have

\[ V = 3000 \cdot 2^{t/9} + 5000 \cdot 2^{t/9} \]
\[ = 8000 \cdot 2^{t/9} \]
\[ = 8000 \left( 2^{1/9} \right)^t \]
\[ = 8000 \cdot 1.08^t, \]

so \( a = 8000, b = 1.08, r = b - 1 = 0.08 = 8\% \).

41. After 12 months it increases by 12 factors of 1.011, or by a combined factor of

\[ 1.011^{12} = 1.1403. \]

Thus, it grows by 14.03\% per year.

42. The value increases by a factor of 1.0006 each day, or by a combined factor of

\[ 1.0006^{365} = 1.2447 \]

each year. This means it grows by 24.47\% each year.

43. The value changes by a factor of 0.995 each week, or by a combined factor of

\[ 0.995^{52} = 0.7705 \]

after one year. Thus, it decreases by 22.95\% after one year.

44. The area is multiplied by a factor of 2/3 every five years, or by a combined factor of

\[ \left( \frac{2}{3} \right)^4 = 0.1975 \]

after twenty years. This means it loses 80.25\% of its area over twenty years.
45. The initial price, $6, is multiplied by 1.05 for each of the 7 years, but the 6 is not raised to the power. Thus,

\[ \text{Price in 7 years time} = 6(1.05)^7. \]

46. A price increasing at 5% a year increases by a factor of 1.05, not 0.05, each year. Thus

\[ \text{Price in 10 years time} = 3(1.05)^{10}. \]

47. Prices will rise by a factor of \((1.05)^{25}\) during the 25 years. We know that

\[ \text{Percent increase} = \text{Growth factor} - 1, \]

so

\[ \text{Percent increase} = (1.05)^{25} - 1. \]

48. At the end of one year, \(t = 12\), so this formula would give \(100(1.05)^{12-12} = 100(1.05)^{144}\) dollars, instead of $100(1.05) as expected. The correct formula is

\[ \text{Price in } t \text{ months} = 100(1.05)^{t/12}. \]

49. If the rate at which prices increase doubles, the factor by which prices change each year is 1.10 instead of 1.05. Since the yearly growth factor is \(1 + r/100\), where \(r\) is the percentage interest rate, doubling the interest rate doubles \(r\) but does not double the 1. Thus, the price of a $20 object in 7 years time is $20(1.10)^7.

50. When \(t = 1\) (that is, after ten years), the formula would give a price of $45(1.05)^{10.10}, instead of $45(1.10)^{10} as expected. The correct formula is

\[ \text{Price in } t \text{ decades} = 45(1.05)^{10t}. \]

51. After 10 years, prices change by a factor of \((1.05)^{10} = 1.629\), so the percentage growth has been \(1.629 - 1 = 0.629 = 62.9\%\). Thus, the formula is

\[ \text{Percentage change over a decade} = ((1.05)^{10} - 1) \cdot 100\%. \]

52. Suppose \(r\) is the percentage change in price each month. Then in 12 months, prices change by a factor of \((1 + r/100)^{12}\). Since we know prices change by 5% in a year, we know that

\[ \left(1 + \frac{r}{100}\right)^{12} = 1.05 \]

\[ 1 + \frac{r}{100} = (1.05)^{1/12} \]

\[ r = ((1.05)^{1/12} - 1)100 \]

\[ \text{Price change per month} = ((1.05)^{1/12} - 1)100\%. \]

Note that \((5/12)\% = 0.417\%\) but \(r = (1.05)^{1/12} - 1 = 0.407\%\).

53. After the budget is trimmed, it is $250(0.99) million. Thus,

\[ \text{Trimmed budget after 10 years} = 250(0.99)(1.05)^{10} \text{ million}. \]

The 1% reduction does not affect the yearly increases of 1.05 but affects the initial quantity.
54. (a) The value of the investment starts at $3000. After $t = 1$ year it increases by a factor of $1.0088^{12}$. This means that it earns 0.88% interest 12 times per year, or 0.88% interest every month.

(b) We have
\[
3000(1.0088)^{12t} = 3000 \left(1.0088^{12}\right)^t \\
= 3000 \cdot 1.1109^t,
\]
so the annual growth rate is 11.09%.

55. (a) The value of the investment starts at $6000. After $t = 1$ year it increases by a factor of $1.021^4$. This means that it earns 2.1% interest 4 times per year, or 2.1% interest every quarter.

(b) We have
\[
6000(1.021)^{4t} = 6000 \left(1.021^4\right)^t \\
= 6000 \cdot 1.0867^t,
\]
so the annual growth rate is 8.67%.

56. (a) The value of the investment starts at $250. After $t = 1$ year it increases by a factor of $1.0011^{365}$. This means that it earns 0.11% interest 365 times per year, or 0.11% interest daily.

(b) We have
\[
250(1.0011)^{365t} = 250 \left(1.0011^{365}\right)^t \\
= 250 \cdot 1.4937^t,
\]
so the annual growth rate is 49.37%.

57. (a) The value of the investment starts at $400. After $T = 1$ month it increases by a factor of $1.007^{1/2}$. This means that it increases by a factor of 1.2 every 2 years, so that it grows by 20% every 2 years.

(b) After 1 year or $T = 12$ months, we have
\[
400(1.007)^{12} = 400 \cdot 1.007^{12} \\
= 400 \cdot 1.0873,
\]
so the annual growth rate is 8.73%.

58. (a) The value of the investment starts at $625. After $T = 3$ months it increases by a factor of $1.03^{1/3} = 1.03$. This means that it earns 3% interest every 3 months, or 3% every quarter.

(b) After 1 year or $T = 12$ months, we have
\[
625(1.03)^{12/3} = 625 \cdot 1.03^4 \\
= 625 \cdot 1.1255,
\]
so the annual growth rate is 12.55%.

59. (a) The value of the investment starts at $500. After $t = 2$ years it increases by a factor of $1.2^{2/2} = 1.2$. This means that it increases by a factor of 1.2 every 2 years, so that it grows by 20% every 2 years.

(b) After $t = 1$ year, we have
\[
500(1.2)^{1/2} = 500 \cdot 1.0954,
\]
so the annual growth rate is 9.54%. 
1. We have

\[ 2^t = 4 \]
\[ 2^t = 2^2 \]
\[ t = 2. \]

2. We have

\[ 4^t = 4 \]
\[ 4^t = 4^1 \]
\[ t = 1. \]

3. We have

\[ 3^t = 1 \]
\[ 3^t = 3^0 \]
\[ t = 0. \]

4. We have

\[ 2g(t) = 162 \]
\[ g(t) = 81 \]
\[ 3^t = 3^4 \]
\[ t = 4. \]

5. We have

\[ 2 + h(t) = \frac{33}{16} \]
\[ h(t) = \frac{33}{16} - 2 \]
\[ h(t) = \frac{33}{16} - \frac{32}{16} \]
\[ h(t) = \frac{1}{16} \]
\[ 4^t = 4^{-2} \]
\[ t = -2. \]

6. We have

\[ 2(1 - f(t)) = 1 \]
\[ 1 - f(t) = \frac{1}{2} \]
\[ -f(t) = \frac{1}{2} \]
\[ f(t) = \frac{1}{2} \]
\[ 2^t = 2^{-1} \]
\[ t = -1. \]
7. We can solve this by noticing that the only power of 2 that equals the same power of 4 is the 0th power, because $2^0 = 4^0 = 1$. This means $f(0) = h(0) = 1$, so a solution is $t = 0$. Algebraically, we write
\[
2^t = 4^t \\
2^t = 2^{2t} \\
t = 2t \\
t = 0.
\]

8. 
\[
h(t) = f(6) \\
4^t = 2^6 \\
= 64 \\
= 4^3 \\
t = 3.
\]

9. From the table we have $t = 2.6$.

10. Multiplying both sides by 2 gives 
\[
12(1.32)^t = 20.9.
\]
From the table, the solution is $t = 2.0$.

11. From the table, $t$ is greater than 2.6 and less than 2.8, that is, $2.6 < t < 2.8$.

12. 
\[
25 - 12(1.32)^t = 2.9 \\
12(1.32)^t = 25 - 2.9 \\
12(1.32)^t = 22.1.
\]
From the table, $t = 2.2$.

13. A positive solution, because 250 is greater than 1.

14. A positive solution, because 2.5 is greater than 1.

15. A negative solution, because 0.3 is smaller than 1.

16. No solution, because $6^t$ is never negative.

PROBLEMS

17. (a) Since $t$ can take on any value, the domain is all real numbers. Since $(0.97)^t$ can never be less than 0, but can get very close to 0 as $t$ gets very large and arbitrarily large as $t$ goes toward large negative numbers, $Q$ is always greater than zero, so the range is $Q > 0$.
(b) Since the lab receives the sample at time $t = 0$, the domain is $t \geq 0$. At $t = 0$, we have $Q = 200(0.97)^0 = 200$, which is the largest value $Q$ can take on. Since $Q > 0$, the range is $0 < Q \leq 200$, meaning that the sample is always between 0 and 200 grams.

18. (a) The amount, $Q$, of an exponentially growing quantity at time $t$ is given by $Q = ab^t$, where $a$ is the initial quantity and $b$ is the growth factor, $b > 1$. If $a = 5$ and $Q = 10$, the equation is
\[
10 = 5b^t.
\]
Equation (II) is of this form with $b = 1.2$. In addition, equation (V) can be transformed into the same form:
\[
10(0.8)^t = 5 \\
10 = 5\left(\frac{1}{0.8}\right)^t = 5\left(\frac{1}{0.8}\right)^t = 5(1.25)^t \\
10 = 5(1.25)^t.
\]
(b) An exponentially decaying quantity is described by $Q = ab^t$, where $a$ is the initial quantity and $b$ is the decay factor, $0 < b < 1$. In this case, $a = 10$ and $Q = 5$, so

$5 = 10b^t$.

Equation (V) is of this form with $b = 0.08$. In addition, equation (II) can be transformed into this form:

$5 = 10 \left( \frac{1}{1.2} \right)^t = 10(0.833)^t$.

19. (a) The balance, $B$, in a bank account at time $t$ is given by $B = B_0b^t$ where $B_0$ is the initial deposit and $b$ is the growth factor, $b > 1$. The balance has doubled when $B = 2B_0$, so

$2B_0 = B_0b^t$

$2 = b^t$.

This is equation (IV), with $b = 1.1$. This is the only answer.

(b) The quantity, $Q$, of a radioactive substance remaining at time $t$ is given by $Q = Q_0b^t$, where $Q_0$ is the initial quantity and $b$ is the decay factor, $0 < b < 1$. The half-life is the solution to

$\frac{1}{2}Q_0 = Q_0b^t$

$\frac{1}{2} = b^t$.

This is equation (V) with $b = 0.9$. This is the only answer.

(c) A quantity growing according to $2^t$ has a doubling time of 1. Equation (III) determines the time it takes such a quantity to quadruple.

20. (a) Reading from the table, we see that $f(t) < 5199$ for $t = 0$, $t = 1$, and $t = 2$. This tells us the balance is less than $5199$ initially and in years $t = 1, 2$.

(b) Reading from the table, we see that $f(t) > 5199$ for $t = 4$. This tells us the balance is greater than $5199$ in year $t = 4$.

(c) Reading from the table, we see that $f(t) = 5199$ at $t = 3$. This tells us the balance reaches $5199$ in year $t = 3$.

21. (a) Evaluating $g(x) = 28(1.1)^x$ gives:

$g(0) = 28(1.1)^0 = 28$
$g(1) = 28(1.1)^1 = 30.8$
$g(2) = 28(1.1)^2 = 33.88$
$g(3) = 28(1.1)^3 = 37.268$
$g(4) = 28(1.1)^4 = 40.995$.

See Table 10.7.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(x)$</td>
<td>28</td>
<td>30.8</td>
<td>33.88</td>
<td>37.268</td>
<td>40.995</td>
</tr>
</tbody>
</table>

(b) (i) In Table 10.7, we see that $g(x) < 33.88$ when $x = 0$ and when $x = 1$.

(ii) In Table 10.7, we see that $g(x) > 30.8$ when $x = 2$, when $x = 3$, and when $x = 4$.

(iii) In Table 10.7, we see that $g(x) = 37.268$ when $x = 3$. 
22. (a) Substituting values for $x$ into $y = 526(0.87)^x$ and evaluating gives:

\[
\begin{align*}
y &= 526(0.87)^{-2} = 694.940 \\
y &= 526(0.87)^{-1} = 604.598 \\
y &= 526(0.87)^0 = 526 \\
y &= 526(0.87)^1 = 457.62 \\
y &= 526(0.87)^2 = 398.129.
\end{align*}
\]

See Table 10.8.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>694.940</td>
<td>604.598</td>
<td>526</td>
<td>457.62</td>
<td>398.129</td>
</tr>
</tbody>
</table>

(b) In Table 10.8, we see that $y = 604.598$ when $x = -1$.

23. (a) Substituting values for $x$ into $y = 253(2.65)^x$ and evaluating gives:

\[
\begin{align*}
y &= 253(2.65)^{-3.5} = 8.351 \\
y &= 253(2.65)^{-1.5} = 58.648 \\
y &= 253(2.65)^{0.5} = 411.854 \\
y &= 253(2.65)^{2.5} = 2892.246.
\end{align*}
\]

See Table 10.9.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3.5</th>
<th>-1.5</th>
<th>0.5</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>8.351</td>
<td>58.648</td>
<td>411.854</td>
<td>2892.246</td>
</tr>
</tbody>
</table>

(b) In Table 10.9, we see that $253(2.65)^x \geq 58.648$ when $x = -1.5$, when $x = 0.5$, and when $x = 2.5$.

24. (a) The table is

<table>
<thead>
<tr>
<th>$t$</th>
<th>55</th>
<th>56</th>
<th>57</th>
<th>58</th>
<th>59</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>9636.03</td>
<td>9751.66</td>
<td>9868.68</td>
<td>9987.11</td>
<td>10106.95</td>
<td>10288.24</td>
</tr>
</tbody>
</table>

(b) Since the doubling time is the number of years it takes for the investment to double, the doubling time is between 58 and 59 years and is closer to 58 years.

25. (a) The table is

<table>
<thead>
<tr>
<th>$t$</th>
<th>3</th>
<th>3.5</th>
<th>3.6</th>
<th>3.7</th>
<th>3.8</th>
<th>3.9</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>57.2</td>
<td>52.1</td>
<td>51.1</td>
<td>50.2</td>
<td>49.3</td>
<td>48.4</td>
<td>47.5</td>
<td>39.4</td>
</tr>
</tbody>
</table>

(b) Since the half-life of caffeine is the time for the 100 mg to be reduced to 50 mg, the half-life is between 3.7 and 3.8 hours and closer to 3.7 hours.
26. (a) We know that \( Q = ab^t \). Since we start with 1000 gm, we know \( a = 1000 \). Since the decay is 7%/day, we know \( b = 1 - 0.07 = 0.93 \). Thus, \( Q = 1000(0.93)^t \).
   (b) Substituting \( t = 8, 9, 10, 11, 12 \) into \( Q = 1000(0.93)^t \) gives the results in Table 10.10.

<table>
<thead>
<tr>
<th>( t )</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>559.582</td>
<td>520.411</td>
<td>483.982</td>
<td>450.104</td>
<td>418.596</td>
</tr>
</tbody>
</table>

(c) (i) Reading from Table 10.10, we see that there are less than 500 gm when \( t = 10 \), when \( t = 11 \), and when \( t = 12 \).
   (ii) Reading from Table 10.10, we see that there are more than 500 gm when \( t = 8 \) and when \( t = 9 \).
(d) The half-life is the amount of time it takes for the substance to decay to half of its original quantity, in this case, to 500 gm. Looking at Table 10.10, we see that this occurs between the ninth and tenth days. The lab worker’s estimate is not consistent with the table.

27. (a) Rounding the numbers to the nearest ant gives Table 10.11.

<table>
<thead>
<tr>
<th>( t )</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>2553</td>
<td>3258</td>
<td>4158</td>
<td>5307</td>
<td>6773</td>
<td>8644</td>
<td>11,032</td>
<td>14,080</td>
</tr>
<tr>
<td>( S )</td>
<td>1611</td>
<td>2594</td>
<td>4177</td>
<td>6727</td>
<td>10,835</td>
<td>17,449</td>
<td>28,102</td>
<td>45,259</td>
</tr>
</tbody>
</table>

(b) We see in Table 10.11 that the small ant colony is close to twice as big as the large ant colony in year 30.
(c) We see in Table 10.11 that there are more large ants than small ants when \( t = 5 \) and when \( t = 10 \). So the large ants harvest the fruit in years 5 and 10.

28. (a) It looks like \( 100(1.05)^t = 121.55 \) when \( t = 4 \), so \( 100(1.05)^t > 121.55 \) when \( t > 4 \).
   (b) It looks like \( 100(1.05)^t = 121.55 \) when \( t = 4 \), so \( 100(1.05)^t < 121.55 \) when \( t < 4 \).

29. (a) On the graph, we can see that the value is greater than $1250 in years 0, 1, and 2. In year 3, the car is worth exactly $1250, so the value is not greater than that $1250.
   (b) The value is given in the graph and by \( 10,000(0.5)^t \). Thus, \( 5000 = 10,000(0.5)^t \) when the car is worth $5000, which occurs in year 1, when \( t = 1 \).

30. (a) See Figure 10.5.
   (b) (i) Looking at Figure 10.5, we see that \( 500(0.8)^x = 608.71 \) when \( x = -1.5 \).
   (ii) Looking at Figure 10.5, we see that \( 500(0.8)^x \geq 400 \) when \( x \leq 1 \).

31. (a) See Figure 10.6.
   (b) (i) Looking at Figure 10.6, we see that \( 250(1.1)^x > 200(1.2)^x \) when \( x \leq 2 \).
   (ii) Looking at Figure 10.6, we see that \( 250(1.1)^x < 200(1.2)^x \) when \( x \geq 3 \).
(c) We could find more values for each equation between \( x = 2 \) and \( x = 3 \), which would give us a more precise answer.

\[ y = 200(1.2)^x \]
\[ y = 250(1.1)^x \]

Figure 10.6

32. (a) The initial (current) salary is 45 thousand dollars, and the salary grows by a factor of 1.041, or 4.1% per year.

(b) The salary in 15 years is given by
\[ 45(1.041)^{15} = 82.2 \]
or 82.2 thousand dollars.

The salary in 20 years is given by
\[ 45(1.041)^{20} = 100.5 \]
or 100.5 thousand dollars.

(c) The salary has doubled when it is $90 thousand, so 15 years is too short and 20 years is too long. Evaluating the salary for the years in between (see Table 10.12) shows that it takes about 17 years for the salary to double.

<table>
<thead>
<tr>
<th>Table 10.12</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(t)</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>45(1.041)^t</td>
<td>85.6</td>
<td>89.1</td>
<td>92.8</td>
</tr>
</tbody>
</table>

33. The equation is equivalent to \( 2^y = -2 \), which has no solution because \( 2^y \) is never negative.

34. The equation is equivalent to \( 3^z = 15/25 = 0.6 \). The solution is negative because 0.6 is smaller than 1.

35. The equation is equivalent to
\[ 13 \cdot \frac{5^{t+1}}{5^t} = \frac{5^{2t}}{5^t} \]
\[ 13 \cdot 5 = 5^t. \]

Since 13 · 5 is greater than 1, the solutions is positive.

36. Since 0.1 is less than 1, negative powers of 0.1 give numbers which are larger than 1. Since 2 > 1, the solution is negative.

37. This equation is equivalent to \( (0.5)^y = 1/5 \). Since 0.5 is less than 1, positive powers of 0.5 give numbers which are smaller than 1. Since 1/5 < 1, the solution is positive.

38. This equation is equivalent to \( (0.7)^t = -5 \), which has no solution because \( (0.7)^t \) is never negative.

39. This equation is equivalent to \( 4 = (0.4)^t \). Since 0.4 is smaller than 1, negative powers of 0.4 give numbers which are larger than 1. Since 4 > 1, the solution is negative.

40. This equation is equivalent to \( 7/28 = (0.4)^t \) or \( (0.4)^t = 1/4 \). Since 0.4 is smaller than 1, positive powers of 0.4 give numbers which are smaller than 1. Since 1/4 < 1, the solution is positive.

41. This equation is equivalent to
\[ \frac{0.01(0.3)^t}{0.01} = \frac{0.1}{0.01} \]
\[ (0.3)^t = 10. \]

Since 0.3 is less than 1, negative powers of 0.3 give numbers which are bigger than 1. Since 10 > 1, the solution is negative.
42. This equation is equivalent to
\[ \frac{10^t}{5^t} = \frac{7}{5} \cdot \frac{5}{5^t}. \]
Since \(10^t/5^t = (10/5)^t = 2^t\), the equation is equivalent to \(2^t = 7\), which has a positive solution.

43. Since \(4^t \cdot 3^t = (4 \cdot 3)^t = 12^t\), this equation is equivalent to \(12^t = 5\), which has a positive solution.

44. The equation is equivalent to
\[ \frac{(3.2)^{2y+1}(4.2)^y}{(3.2)^y} = \frac{(3.2)^y}{(3.2)^y} \]
\[ (3.2)^{y+1}(4.2) = 1 \]
\[ (3.2)^y(3.2)(4.2) = 1 \]
\[ (3.2)^y = \frac{1}{(3.2)(4.2)}. \]
Since \(1/(3.2)(4.2)\) is smaller than 1, the solution is negative.

45. In the equation \(P_0(1+r)^x = P\), we think of \(x\) as the time it takes for a quantity \(P_0\) to grow to \(P\). Using this interpretation, in (I), we think of \(x\) as the time it takes for 3 to grow to 7 at a growth rate of \(r\). In (II), the growth rate is doubled, so the time decreases from (I) to (II). In (III), the growth rate is decreased, so the time increases from (I) to (III). In (IV), the growth rate is negative, so the quantity \(P\) decreases with time. Since 7 is larger than 3, there is no solution to (IV).

Thus, (III) has the largest solution; (II) has the smallest solution; (IV) has no solution.

46. (a) As \(a\) increases, \(x\) decreases.
(b) As \(r\) increases, \(x\) decreases.
(c) As \(b\) increases, \(x\) increases.

47. (a) The equation has a solution if \(A > 0\).
(b) The equation has solution \(t = 0\) if \(A = 5^0 = 1\).
(c) Since \(5^t\) is greater than 1 if \(t\) is positive, the equation has a positive solution if \(A > 1\).

48. The equation is equivalent to
\[ \frac{1}{3^t} = A \]
\[ 3^t = \frac{1}{A} \]

(a) The equation has a solution if \(1/A\) is positive, that is, if \(A > 0\).
(b) The equation has solution \(t = 0\) if
\[ 3^0 = \frac{1}{A} \]
\[ A = 1. \]

(c) Since \(3^t\) is greater than 1 if \(t\) is positive, the equation has positive solution if
\[ \frac{1}{A} > 1 \]
\[ A < 1. \]

Since \(A > 0\) if there is any solution, we have \(0 < A < 1\).

49. (a) The equation has a solution if \(A > 0\).
(b) The equation has solution \(t = 0\) if \(A = (0.2)^0 = 1\).
(c) Since \((0.2)^t\) is smaller than 1 if \(t\) is positive, the equation has a positive solution if \(A < 1\). Thus, we have \(0 < A < 1\).
50. This equation is equivalent to
\[ A = 2^{-t} = \frac{1}{2^t} \]
\[ 2^t A = 1 \]
\[ 2^t = \frac{1}{A} \]

(a) The equation has a solution if \( 1/A \) is positive, that is, if \( A > 0 \).
(b) The equation has solution \( t = 0 \) if
\[ 2^0 = \frac{1}{A} \]
\[ A = 1. \]
(c) Since \( 2^t \) is greater than 1 if \( t \) is positive, the equation has a positive solution if
\[ \frac{1}{A} > 1 \]
\[ A < 1. \]

Since we know \( A > 0 \), the equation has a positive solution for \( 0 < A < 1 \).

51. This equation is equivalent to
\[ 6.3A = 3 \cdot 7^t \]
\[ 7^t = \frac{6.3A}{3} = 2.1A. \]

(a) The equation has a solution if \( 2.1A \) is positive, that is, if \( A > 0 \).
(b) The equation has solution \( t = 0 \) if
\[ 7^0 = 2.1A \]
\[ A = \frac{1}{2.1}. \]
(c) Since \( 7^t \) is greater than 1 if \( t \) is positive, the equation has a positive solution if
\[ 2.1A > 1 \]
\[ A > \frac{1}{2.1}. \]

52. The equation is equivalent to
\[ 2 \cdot 3^t = -A \]
\[ 3^t = -\frac{A}{2}. \]

(a) The equation has a solution if \( -A/2 \) is positive, that is, if \( A < 0 \).
(b) The equation has solution \( t = 0 \) if
\[ 3^0 = -\frac{A}{2} \]
\[ A = -2. \]
(c) If \( t \) is positive, \( 3^t \) is greater than 1. Thus
\[ -\frac{A}{2} > 1 \]
\[ -A > 2 \]
\[ A < -2. \]
53. The equation is equivalent to

\[ A \cdot 5^{-t} = -1 \]
\[ A \cdot \frac{1}{5^t} = -1 \]
\[ 5^t = -A. \]

(a) The equation has a solution if \(-A\) is positive, that is, if \(A < 0\).
(b) The equation has solution \(t = 0\) if

\[ 5^0 = -A \]
\[ A = -1. \]

(c) Since \(5^t\) is greater than 1 if \(t\) is positive, the equation has a positive solution if

\[ -A > 1 \]
\[ A < -1. \]

54. The equation is equivalent to

\[ 2(0.7)^t = -0.2A \]
\[ (0.7)^t = -0.1A. \]

(a) The equation has a solution if \(-0.1A\) is positive, that is, if \(A < 0\).
(b) The equation has solution \(t = 0\) if

\[ (0.7)^0 = -0.1A \]
\[ A = -\frac{1}{0.1} = -10. \]

(c) Since \((0.7)^t\) is smaller than 1 if \(t\) is positive, the equation has a positive solution if

\[ -0.1A < 1 \]
\[ A > -\frac{1}{0.1} = -10. \]

Since \(A < 0\) for any solution, we have \(-10 < A < 0\).

55. We have

\[ 10^x = 1000^u \]
\[ 10^x = (10^3)^u \]
\[ 10^x = 10^{3u} \]
\[ x = 3u. \]

56. We have

\[ 8^x = 2^u \]
\[ (2^3)^x = 2^u \]
\[ 2^{3x} = 2^u \]
\[ 3x = u \]
\[ x = \frac{1}{3}u. \]
57. We have
\[ 5^{2x+1} = 125^u \]
\[ 5^{2x+1} = (5^3)^u \]
\[ 5^{2x+1} = 5^{3u} \]

\[ 2x + 1 = 3u \]
\[ x = \frac{1}{2}(3u - 1). \]

58. We have
\[ \left( \frac{1}{2} \right)^x = \left( \frac{1}{16} \right)^u \]
\[ \left( \frac{1}{2} \right)^x = \left( \frac{1}{2} \right)^4 \]
\[ \left( \frac{1}{2} \right)^x = \left( \frac{1}{2} \right)^{4u} \]
\[ x = 4u. \]

59. We have
\[ \left( \frac{1}{3} \right)^x = 9^u \]
\[ (3^{-1})^x = (3^2)^u \]
\[ 3^{-x} = 3^{2u} \]
\[ -x = 2u \]
\[ x = -2u. \]

60. We have
\[ 5^{-x} = \frac{1}{25}^u \]
\[ 5^{-x} = \left( \frac{1}{25} \right)^u \]
\[ 5^{-x} = (5^{-2})^u \]
\[ 5^{-x} = 5^{-2u} \]
\[ -x = -2u \]
\[ x = 2u. \]

61. We have
\[ 10^x = 15 \]
\[ = 3 \cdot 5 \]
\[ = 10^{0.477} \cdot 10^{0.699} \]
\[ = 10^{0.477} \cdot 0.699 \]
\[ = 10^{1.176}, \]

so \( x = 1.176. \)
62. We have
\[10^x = 9\]
\[= 3 \cdot 3\]
\[= 10^{0.477} \cdot 10^{0.477}\]
\[= 10^{0.477 + 0.477}\]
\[= 10^{0.954},\]
so \(x = 0.954\).

63. We have
\[10^x = 32\]
\[= 2^5\]
\[= (10^{0.301})^5\]
\[= 10^{5 \cdot 0.301}\]
\[= 10^{1.505},\]
so \(x = 1.505\).

64. We have
\[10^x = 180\]
\[= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5\]
\[= 10^{0.301} \cdot 10^{0.301} \cdot 10^{0.477} \cdot 10^{0.477} \cdot 10^{0.699}\]
\[= 10^{0.301 + 0.301 + 0.477 + 0.477 + 0.699}\]
\[= 10^{2.255},\]
so \(x = 2.255\).

Solutions for Section 10.5

EXERCISES

1. Let \(Q = Q_0a^t\), where \(t\) is in years. Then \(Q = 2Q_0\) when \(t = 9\), so
\[2Q_0 = Q_0a^9\]
\[a^9 = 2\]
\[a = 2^{1/9} = 1.08.\]
Thus,
\[Q = Q_0(1.08)^t.\]

2. Let \(Q = Q_0a^t\), where \(t\) is in days. Then \(Q = (1/2)Q_0\) when \(t = 5\), so
\[\frac{1}{2}Q_0 = Q_0a^5\]
\[a^5 = \frac{1}{2}\]
\[a = \left(\frac{1}{2}\right)^{1/5} = 0.87.\]
Thus,
\[ Q = Q_0(0.87)^t. \]

3. Let \( B = B_0a^t \), where \( t \) is in years. Then \( B = 3B_0 \) when \( t = 20 \), so
\[
\begin{align*}
3B_0 &= B_0a^{20} \\
3 &= a^{20} \\
a &= 3^{1/20} = 1.056.
\end{align*}
\]
Thus,
\[ B = B_0(1.056)^t. \]

4. We let the initial population be \( a \) and call the growth factor per year \( b \), so \( ab^t \) gives the population after \( t \) years. After 50 years, the population is \( ab^{50} \), and since 50 years is the doubling time, \( ab^{50} \) is also equal to twice the initial population, of \( 2a \). Thus
\[
\begin{align*}
ab^{50} &= 2a \\
b^{50} &= 2 \\
b &= 2^{1/50} = 1.014.
\end{align*}
\]
Thus, the population is multiplied by 1.014 each year, giving a growth rate of 1.4%.

5. We let the initial population be \( a \) and call the growth factor per year \( b \), so \( ab^t \) gives the population after \( t \) years. After 143 years, the population is \( ab^{143} \), and since 143 years is the doubling time, \( ab^{143} \) is also equal to twice the initial population, of \( 2a \). Thus
\[
\begin{align*}
ab^{143} &= 2a \\
b^{143} &= 2 \\
b &= 2^{1/143} = 1.00486.
\end{align*}
\]
Thus, the population is multiplied by 1.00486 each year, giving a growth rate of 0.486%.

6. We let the initial population be \( a \) and call the growth factor per year \( b \), so \( ab^t \) gives the population after \( t \) years. After 14 years, the population is \( ab^{14} \), and since 14 years is the doubling time, \( ab^{14} \) is also equal to twice the initial population, of \( 2a \). Thus
\[
\begin{align*}
ab^{14} &= 2a \\
b^{14} &= 2 \\
b &= 2^{1/14} = 1.0508.
\end{align*}
\]
Thus, the population is multiplied by 1.0508 each year, giving a growth rate of 5.08%.

7. The value, \( V \), of the investment is growing exponentially, so with \( t \) in years,
\[ V = ab^t. \]
The initial value is \( a \), so we know \( V = 2a \) when \( t = 7 \):
\[ 2a = ab^7. \]
Canceling gives
\[ b^7 = 2, \]
so taking the \( 1/7 \) power of each side
\[ b = 2^{1/7} = 1.10409. \]
Thus, the growth rate is 10.400% per year.
8. The quantity, \( n \) of nicotine at time \( t \) in hours is given by

\[
n = ab^t.
\]

The initial value is \( a \), so we know \( n = a/2 \) when \( t = 2 \):

\[
\frac{a}{2} = ab^2.
\]

Canceling gives

\[
\frac{1}{2} = b^2,
\]

so, taking square roots,

\[
b = \sqrt{\frac{1}{2}} = 0.70711.
\]

Thus, at the end of each hour, 70.711\% remains, so 29.289\% has decayed.

9. Let \( a \) be the initial amount of bismuth-210 and let \( y \) be the amount remaining after \( t \) days. Then \( y = ab^t \). When \( t = 5 \) days we have \( y = a/2 \). So we get \( a/2 = ab^5 \). This gives \( 0.5 = b^5 \), so \( b = (0.5)^{1/5} = 0.8706 \). This means 87.06\% of the initial amount remains after 1 day. So the decay rate per day is \( 1 - 0.8706 = 0.1294 \). Thus bismuth-210 decays at 12.94\% per day.

10.

\[
P_0b^8 = 2P_0
\]

\[
b^8 = 2
\]

\[
b = 2^{1/8}
\]

\[
= 1.0905,
\]

so \( r = 0.0905 = 9.05\% \).

11.

\[
V_0b^{14} = 3V_0
\]

\[
b^{14} = 3
\]

\[
b = 3^{1/14}
\]

\[
= 1.0816,
\]

so \( r = 0.0816 = 8.16\% \).

12.

\[
P_0b^7 = 0.5P_0
\]

\[
b^7 = 0.5
\]

\[
b = 0.5^{1/7}
\]

\[
= 0.9057,
\]

so \( r = -0.0943 = -9.43\% \).

13.

\[
V_0b^4 = \frac{1}{5}V_0
\]

\[
b^4 = 0.2
\]

\[
b = 0.2^{1/4}
\]

\[
= 0.6687,
\]

so \( r = -0.3313 = -33.13\% \).
14. \[Q_0 b^5 = 1.25Q_0 \]
\[b^5 = 1.25 \]
\[b = 1.25^{1/5} \]
\[= 1.04564,\]
so \( r = 0.0456 = 4.56\% \).

15. \[Q_0 b^8 = 0.76Q_0 \]
\[b^8 = 0.76 \]
\[b = 0.76^{1/8} \]
\[= 0.9663,\]
so \( r = -0.0337 = -3.37\% \).

16. The starting value is \( a = 40 \), so \( f(t) = 40b^t \). We know that \( f(8) = 100 \) so
\[40b^8 = 100 \]
\[b^8 = \frac{100}{40} = 2.5 \]
\[b = 2.5^{1/8} \]
\[= 1.1214,\]
so \( f(t) = 40(1.1214)^t \).

17. The starting value is \( a = 80 \), so \( g(t) = 80b^t \). We know that \( g(25) = 10 \) so
\[80b^{25} = 10 \]
\[b^{25} = \frac{10}{80} = 0.125 \]
\[b = 0.125^{1/25} = 0.9202,\]
so \( g(t) = 80(0.9202)^t \).

18. We have \( p(5) = 20 \) and \( p(35) = 60 \). Taking ratios gives
\[\frac{ab^{35}}{ab^{5}} = \frac{p(35)}{p(5)} \]
\[b^{30} = \frac{60}{20} \]
\[b = \left(\frac{60}{20}\right)^{1/30} \]
\[= 1.0373.\]

Now we can solve for \( a \):
\[a(1.0373)^5 = 20 \]
\[a = \frac{20}{(1.0373)^5} \]
\[= 16.6536,\]
so \( p(x) = 16.6536(1.0373)^x \).
19. We have \( p(-12) = 72 \) and \( p(6) = 8 \). Taking ratios gives

\[
\frac{ab^6}{ab^{-12}} = \frac{p(6)}{p(-12)}
\]

\[
b^{18} = \frac{8}{72}
\]

\[
b = \left(\frac{8}{72}\right)^{1/18} = 0.8851.
\]

Now we can solve for \( a \):

\[
a(0.8851)^{-12} = 72
\]

\[
a = \frac{72}{(0.8851)^{-12}} = 16.6433,
\]

so \( p(x) = 16.6433(0.8851)^x \).

20. We have \( v(0.1) = 2 \) and \( v(0.9) = 8 \). Taking ratios gives

\[
\frac{ab^{0.9}}{ab^{0.1}} = \frac{v(0.9)}{v(0.1)}
\]

\[
b^{0.8} = \frac{8}{2}
\]

\[
b = \left(\frac{8}{2}\right)^{1/0.8} = 5.6569.
\]

Now we can solve for \( a \):

\[
a(5.6569)^{0.1} = 2
\]

\[
a = \frac{2}{(5.6569)^{0.1}} = 1.6818,
\]

so \( v(x) = 1.6818(5.6569)^x \).

21. We have \( w(5) = 20 \) and \( w(20) = 5 \). Taking ratios gives

\[
\frac{ab^{20}}{ab^{5}} = \frac{w(20)}{w(5)}
\]

\[
b^{15} = \frac{5}{20}
\]

\[
b = \left(\frac{5}{20}\right)^{1/15} = 0.9117.
\]

Now we can solve for \( a \):

\[
a(0.9117)^5 = 20
\]

\[
a = \frac{20}{(0.9117)^5} = 31.752,
\]

so \( w(x) = 31.752(0.9117)^x \).
22. We need only two points to find $a$ and $b$. Using the first two points we have
\[
ab^2 = 96 \\
ab^3 = 76.8.
\]
Dividing the two equations gives
\[
\frac{ab^3}{ab^2} = \frac{76.8}{96}.
\]
The left-hand side of the equation simplifies to $b$, so we have
\[
b = 0.8.
\]
Then we solve for $a$
\[
a(0.8)^2 = 96 \\
a = \frac{96}{(0.8)^2} = 150.
\]
Thus, the equation is $f(t) = 150(0.8)^t$.

**PROBLEMS**

23. We know the initial value is $a = g(0) = 8000$. We can use the value in year $t = 12$ to solve for the base $b$:
\[
8000b^{12} = 3000 \\
g^{12} = \frac{3}{8} \\
b = \left(\frac{3}{8}\right)^{1/12} \\
= 0.9215.
\]
We have $g(t) = 8000(0.9215)^t$.

24. One approach is to notice that the value of $y$ is halved from 120 to 60, then from 60 to 30, while the value of $x$ climbs from 24 to 48, then from 48 to 72. Using the fact that the half-life is 24, we infer that the starting value at $x = 0$ must be $y = 240$. Using what we know about half-lives, we conclude that
\[
f(x) = 240 \left(\frac{1}{2}\right)^{x/24}.
\]
The above approach may not be apparent right away because the intermediate point $(48, 60)$ and the initial value $(0, 240)$ are not given directly. So another approach is to solve for $b$, then $a$: Since $f(24) = 120$ and $f(72) = 30$, the ratio method gives
\[
\frac{ab^{72}}{ab^{24}} = \frac{f(72)}{f(24)} \\
b^{48} = \frac{30}{120} = \frac{1}{4} \\
b = \left(\frac{30}{120}\right)^{1/48} = 0.9715.
\]
Now we can solve for $a$:
\[
a(0.9715)^{24} = 120 \\
a = \frac{120}{(0.9715)^{24}} = 240,
\]
so \( y = 240(0.9715)^x \). This is the same answer as before, though in a different form, because
\[
240 \left( \frac{1}{2} \right)^{t/24} = 240 \left( \left( \frac{1}{2} \right)^{1/24} \right)^t = 240(0.9715)^t.
\]

25. The starting value is \( a = 3450 \), the growth rate is \( r = 4.75\% \), and the graph factor is \( b = 1 + r = 1.0475 \), so \( V = 3450(1.0475)^t \).

26. We have \( f(5) = 400 \) and \( f(12) = 1200 \), so:
\[
\frac{ab^{12}}{ab^7} = \frac{f(12)}{f(5)},
\]
\[
b^7 = \frac{1200}{400},
\]
\[
b = \left( \frac{1200}{400} \right)^{1/7} = 1.169931.
\]
Now we can solve for \( a \):
\[
a(1.169931)^5 = 400
\]
\[
a = \frac{400}{(1.169931)^5} = 182.498,
\]
so \( V = 182.498(1.169931)^t \).

27. The starting value is \( a = 800 \) and the half-life is 19 days, so one possible formula is
\[
P = 800 \left( \frac{1}{2} \right)^{t/19}.
\]
Another possibility is to write this in standard form:
\[
P = 800 \left( \left( \frac{1}{2} \right)^{1/19} \right)^t
\]
\[
= 800(0.9642)^t.
\]

28. We are given the initial value \( b = 3500 \) and the doubling time of 8 years. This means
\[
V = 3500 \cdot 2^{t/8}.
\]

29. We have \( f(12) = 290 \) and \( f(23) = 175 \), so
\[
\frac{ab^{23}}{ab^{12}} = \frac{f(23)}{f(12)}
\]
\[
b^{11} = \frac{175}{290}
\]
\[
b = \left( \frac{175}{290} \right)^{1/11} = 0.9551205.
\]
Now we can solve for \( a \):
\[
a(0.9551205)^{12} = 290
\]
\[
a = \frac{290}{(0.9551205)^{12}} = 503.153,
\]
so \( f(x) = 503.153(0.9551205)^x \).
30. We have \( f(10) = 65 \) and \( f(65) = 20 \). Taking ratios gives
\[
\frac{ab^{65}}{ab^{10}} = \frac{f(65)}{f(10)}
\]
\[
b^{55} = \frac{20}{65}
\]
\[
b = \left(\frac{20}{65}\right)^{1/55}
\]
\[
= 0.9788.
\]
Now we can solve for \( a \):
\[
a(0.9788)^{10} = 65
\]
\[
a = \frac{65}{(0.9788)^{10}}
\]
\[
= 80.533,
\]
so \( f(x) = 80.533(0.9788)^x \).

31. If \( V \) is the value of the investment and \( t \) is the number of years since the initial deposit, then we have \( V = 3000b^t \). We know that in year \( t = 5 \), \( V = 3000(1.3) = 3900 \), so
\[
3000b^5 = 3000(1.3)
\]
\[
b^5 = 1.3
\]
\[
b = 1.3^{1/5}
\]
\[
= 1.0539,
\]
so \( V = 3000 \cdot 1.0539^t \).

32. If \( P \) is the population and \( t \) is the number of years since it was 10,000, we have \( a = 10,000, r = -0.0042, b = 1 + r = 0.9958 \), so \( P = 10,000 \cdot 0.9958^t \).

33. The graph of \( g \) contains the points \((10, 70)\) and \((80, 20)\). We have \( g(10) = 70 \) and \( g(80) = 20 \). Taking ratios gives
\[
\frac{ab^{80}}{ab^{10}} = \frac{g(80)}{g(10)}
\]
\[
b^{70} = \frac{20}{70}
\]
\[
b = \left(\frac{20}{70}\right)^{1/70}
\]
\[
= 0.9823.
\]
Now we can solve for \( a \):
\[
a(0.9823)^{10} = 70
\]
\[
a = \frac{70}{(0.9823)^{10}}
\]
\[
= 83.687,
\]
so \( g(x) = 83.687(0.9823)^x \).

34. (a) Since the balance doubles in 7 years, it doubles again in the next 7 years. Thus, in 14 years, it is multiplied by a factor of 4. In the third 7-year period, the balance doubles again. Thus, it is multiplied by a factor of 8 in 21 years.

(b) Since we know that \( P = P_0a^t \), where \( P = 2P_0 \) when \( t = 7 \), we have
\[
2P_0 = P_0a^7
\]
\[
a^7 = 2
\]
\[
a = 2^{1/7} = 1.104. 
\]
35. (a) After 62 days, half of the substance decays, leaving \( \frac{1}{2}Q_0 \) or 0.5\( Q_0 \). After 62 more days, or 124 days total, half of this remaining amount decays, leaving \( \frac{1}{4}Q_0 = \frac{1}{4}Q_0 \) or 0.25\( Q_0 \). See Figure 10.7.

(b) In part (a), we saw that or 0.25\( Q_0 \) remains after 124 days. After an additional 62 days, or 186 days total, half of this, or 0.125\( Q_0 \) remains. This is 12.5\% of the original amount. See Figure 10.7.

(c) Let \( w(t) = ab^t \) give the amount of substance after \( t \) days. Since the initial amount is \( Q_0 \), we have \( a = Q_0 \). After 62 days, the amount remaining is 0.5\( Q_0 \), so:

\[
\begin{align*}
\frac{w(62)}{Q_0} &= 0.5 \\
Q_0b^{62} &= 0.5Q_0 \\
b^{62} &= 0.5\\
(b^{62})^{1/62} &= 0.5^{1/62} \\
0.9889 &= b.
\end{align*}
\]

Thus, after 1 day, the amount remaining is

\[
\frac{w(1)}{Q_0} = Q_0(0.9889)^1 = 0.9889Q_0.
\]

This is 98.89\% of the original amount.

![Figure 10.7: Quantity of radioactive substance remaining](image)

36. (a) Using an exponential growth model, with the annual rate of growth 5.3\% and the initial amount 936.7, we get \( P = 936.7(1.053)^t \) billion dollars.

(b) Since \( t = 0 \) corresponds to the year 1994, the year 1999 corresponds to the value of \( t = 5 \). Therefore, \( 936.7(1.053)^5 = 1212.7 \) billion dollars. This estimate is about 2 billion dollars off the actual value of 1210.7 billion dollars.

37. (a) Let \( t \) be years since 1990. Then if \( P \) is the population

\[
P = 2.8b^t
\]

We know \( P = 2.9 \) when \( t = 10 \), so

\[
2.9 = 2.8b^{10} \\
b^{10} = \frac{2.9}{2.8} = 1.035714.
\]

Taking tenth roots of both sides gives

\[
b = (1.035714)^{1/10} = 1.003515.
\]

Thus, the growth rate is 0.352\% per year.

(b) In 2010, we have \( t = 20 \), so

\[
P = 2.8(1.003515)^{20} = 3.004 \text{ million}.
\]
38. (a) Since $P(0) = 396$, we know that the population of Charlotte was 396,000 when $t = 0$, that is, in 1990. Similarly, $P(10) = 541$ tells us that the population was 541,000 when $t = 0$, that is, in 2000.

(b) The population grows at a constant percentage rate, so $P(t) = ab^t$. Since $P(0) = 396$, we have $a = 396$, so

$$P(t) = 396 \cdot b^t.$$ 

To find $b$, we use the fact that $P(10) = 541$, so

$$541 = 396 \cdot b^{10}$$ 

$$b^{10} = \frac{541}{396} = 1.3662.$$ 

To solve for $b$, we raise both sides to the $1/10$ power,

$$b = \left(\frac{541}{396}\right)^{1/10} = 1.03169.$$ 

Thus

$$P(t) = 396(1.03169)^t.$$ 

(c) Substituting $t = 20$ gives

$$P(20) = 396(1.03169)^{20} \approx 739.1.$$ 

The population when $t = 20$, in 2010, is predicted to be 739,100.

39. (a) Since nicotine leaves the body at a constant percent rate, we know $n(t)$ decays exponentially, so $n(t) = ab^t$.

We know $n(2) = 58$ and $n(5) = 20$, so

$$58 = ab^2$$ 

$$20 = ab^5.$$ 

Dividing the second equation by the first gives

$$\frac{20}{58} = \frac{ab^5}{ab^2}.$$ 

Canceling, we have

$$\frac{20}{58} = b^3,$$ 

so

$$b = \left(\frac{20}{58}\right)^{1/3} \approx 0.70124.$$ 

Thus, at the end of each hour, 70.124% of the material remains, so 29.856% has decayed.

(b) To find the initial quantity, $a$, we solve the equation

$$58 = a \cdot (0.70124)^2$$ 

$$a = \frac{58}{(0.70124)^2} \approx 117.949 \text{ mg}.$$ 

(c) Six hours after the cigarette,

$$\text{Nicotine remaining} = n(6) = 117.949(0.70124)^6 \approx 14.025 \text{ mg}.$$
EXERCISES

1. We have

\[ a = 1200 \quad \text{Starting value is$1200} \]
\[ k = 0.041 \quad \text{Continuous growth rate is 4.1\%} \]
\[ b = e^k = 1.0419 \quad \text{Annual growth factor is 1.0419} \]
\[ r = b - 1 = 0.0419 \quad \text{Annual growth rate is 4.19\%}. \]

2. We have

\[ a = 3500 \quad \text{Starting value is$3500} \]
\[ k = 0.173 \quad \text{Continuous growth rate is 17.3\%} \]
\[ b = e^k = 1.1889 \quad \text{Annual growth factor is 1.1889} \]
\[ r = b - 1 = 0.1889 \quad \text{Annual growth rate is 18.89\%}. \]

3. We have

\[ a = 7500 \quad \text{Starting value is$7500} \]
\[ k = -0.059 \quad \text{Continuous growth rate is -5.9\%} \]
\[ b = e^k = 0.9427 \quad \text{Annual growth factor is 0.9427} \]
\[ r = b - 1 = -0.0573 \quad \text{Annual growth rate is -5.73\%}. \]

4. We have

\[ a = 17,000 \quad \text{Starting value is$17,000} \]
\[ k = 0.322 \quad \text{Continuous growth rate is 32.2\%} \]
\[ b = e^k = 1.3799 \quad \text{Annual growth factor is 1.3799} \]
\[ r = b - 1 = 0.3799 \quad \text{Annual growth rate is 37.99\%}. \]

5. We have

\[ a = 20,000 \quad \text{Starting value is$20,000} \]
\[ k = -0.44 \quad \text{Continuous growth rate is -44\%} \]
\[ b = e^k = 0.6440 \quad \text{Annual growth factor is 0.6440} \]
\[ r = b - 1 = -0.3560 \quad \text{Annual growth rate is -35.6\%}. \]

6. We have

\[ a = 1800 \quad \text{Starting value is$1800} \]
\[ k = 1.21 \quad \text{Continuous growth rate is 121\%} \]
\[ b = e^k = 3.353 \quad \text{Annual growth factor is 3.353} \]
\[ r = b - 1 = 2.353 \quad \text{Annual growth rate is 235.3\%}. \]

7. We have

\[ Q = 20e^{-t/5}, \]
\[ Q = 20e^{-0.2t}, \]

so \( a = 20, k = -0.2 \).
8. We have

\[ Q = 200e^{0.5t-3} \]
\[ = 200e^{0.5t}e^{-3} \]
\[ = (200e^{-3})e^{0.5t} \]
\[ Q = 9.957e^{0.5t}, \]

so \( a = 200e^{-3} = 9.957 \) and \( k = 0.5 \).

9. We have

\[ Q = \frac{1}{40}e^{0.4t} \]
\[ = \frac{1}{40} \cdot \frac{1}{e^{-0.4t}} \]
\[ = 0.025 \cdot (e^{0.4t})^{-1} \]
\[ Q = 0.025e^{-0.4t}, \]

so \( a = 0.025 \), \( k = -0.4 \).

10. We have

\[ Q = 37.5 \cdot (e^{-3t})^2 \]
\[ = 37.5e^{2(-3t)} \]
\[ = 37.5e^{-6t} \]
\[ = (37.5e^2) e^{-6t} \]
\[ Q = 277.09e^{-6t}, \]

so \( a = 37.5e^2 = 277.09 \) and \( k = -6 \).

11. We have

\[ Q = \frac{e^t e^{2t}}{e^{1t}} \]
\[ = e^t \cdot \frac{e^{2t}}{e^{1t}} \]
\[ = e^t \cdot e^{2t-1t} \]
\[ = e^t \cdot e^{-5t} \]
\[ Q = 23.1407e^{-5t}, \]

so \( a = e^t = 23.1407 \), \( k = -5 \).

12. We have

\[ Q = 90\sqrt{e^{-0.4t}} \]
\[ = 90 \cdot (e^{-0.4t})^{0.5} \]
\[ = 90e^{0.5(-0.4t)} \]
\[ Q = 90e^{-0.2t}, \]

so \( a = 90 \), \( k = -0.2 \).

PROBLEMS

13. Both investments have a starting value of \( a = 1000 \). Investment \( A \) has a growth rate of \( r = 10\% \), so \( b = 1 + r = 1.10 \), and

Investment \( A = 1000(1.10)^t \).
Investment $B$ has a continuous growth rate of $k = 10\%$, so

$$Investment \ B = 1000e^{0.10t}.$$  

From Table 10.13, we see that the two investments are roughly equal at first, but that after 50 years investment $B$ is worth quite a bit more—over $31,000 more. This is because

$$e^{0.1} = 1.1052,$$

so 10\% continuous growth corresponds to an annual percent rate of 10.52\%. Thus, investment $B$ grows at a faster rate, and after 50 years this makes a considerable difference.

<table>
<thead>
<tr>
<th>$t$ (yr)</th>
<th>Inv A, $</th>
<th>Inv B, $</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,000.00</td>
<td>1,000.00</td>
</tr>
<tr>
<td>10</td>
<td>2,593.74</td>
<td>2,718.28</td>
</tr>
<tr>
<td>20</td>
<td>6,727.50</td>
<td>7,389.06</td>
</tr>
<tr>
<td>30</td>
<td>17,449.40</td>
<td>20,085.54</td>
</tr>
<tr>
<td>40</td>
<td>45,259.26</td>
<td>54,598.15</td>
</tr>
<tr>
<td>50</td>
<td>117,390.85</td>
<td>148,413.16</td>
</tr>
</tbody>
</table>

14. After $t$ years the population is $15,000e^{0.02t}$, so after 20 years it is $15,000e^{0.02 \cdot 20} = 22,377$, or about 22,400.

15. After $t$ years it is worth $350,000e^{-0.09t}$, so after 5 years it is worth $350,000e^{-0.09 \cdot 5} = 223,169.85$, or about $223,170$.

16. $B(t) = 3500e^{0.15t}$

17. We have

$$V = A e^{-t/(150,000 \cdot 0.0004)} = A e^{-t/60} = A e^{-0.0167t}.$$  

Since the exponent is negative, the voltage is decaying. Thus continuous rate is $-1.67\%$.

18. The starting population is $P(0) = 4000e^{r \cdot 0} = 4000 \cdot e^0 = 4000$

19. $P(0) = P_0e^{0.37 \cdot 0} = P_0 \cdot 1 = P_0$

20. $P(0) = \frac{1500e^0}{30+20} = \frac{1500}{50} = 30$

21. $P(0) = \frac{L}{P_0 + Ae^0} = \frac{L}{P_0 + A}$

22. To eliminate the $e^{kt}$ in the numerator, we divide the numerator and denominator by $e^{kt}$:

$$P = \frac{1500e^{kt}}{30 + 20e^{kt}} = \frac{1500}{30e^{-kt} + 20} = \frac{1500}{20 + 30e^{-kt}}.$$  

Comparing with the form

$$P = \frac{L}{P_0 + Ae^{-kt}},$$

we have $L = 1500$, $P_0 = 20$, and $A = 30$.

23. We have

$$5e^{2+3t} = 5e^2 \cdot e^{3t} = 36.945e^{3t},$$

so $a = 5e^2 = 36.945$, $k = 3$. 


24. We have
\[
50 - \frac{50}{e^{0.25t}} = 50 - 50 \cdot \frac{1}{e^{0.25t}} = 50 - 50 \left( e^{0.25t} \right)^{-1} = 50 - 50e^{-0.25t} = 50 \left( 1 - e^{-0.25t} \right),
\]
so \( A = 50, r = 0.25 \).

25. We have
\[
\left( e^{3x} + 2 \right)^2 = \left( e^{3x} + 2 \right) \left( e^{3x} + 2 \right) = e^{3x} \cdot e^{3x} + 2e^{3x} + 2e^{3x} + 4 = e^{3x+3x} + 4e^{3x} + 4 = e^{6x} + 4e^{3x} + 4,
\]
so \( r = 6, a = 4, s = 3, b = 4 \).

26. We have
\[
\frac{e^{0.1t}}{2e^{0.1t} + 3} = \frac{e^{0.1t}}{2e^{0.1t} + 3} \cdot \frac{e^{-0.1t}}{e^{-0.1t}} = \frac{e^{0.1t} \cdot e^{-0.1t}}{(2e^{0.1t} + 3) e^{-0.1t}} = \frac{1}{2e^{0.1t} \cdot e^{-0.1t} + 3 \cdot e^{-0.1t}} = \frac{1}{2 + 3 \cdot e^{-0.1t}},
\]
so \( a = 2, b = 3, k = 0.1 \).

27. Breaking the expression into two parts, we have:
\[
(cosh x)^2 = \left( \frac{e^x + e^{-x}}{2} \right)^2 = \frac{e^x + e^{-x}}{2} \cdot \frac{e^x + e^{-x}}{2} = \frac{e^{2x}}{e^x \cdot e^x + e^{-x} \cdot e^{-x} + e^{-x} \cdot e^x + e^{-x} \cdot e^{-x}} = \frac{e^{2x} + 2 + e^{-2x}}{4} = \frac{e^{2x} - 2 + e^{-2x}}{4},
\]
\[
(sinh x)^2 = \left( \frac{e^x - e^{-x}}{2} \right)^2 = \frac{e^x - e^{-x}}{2} \cdot \frac{e^x - e^{-x}}{2} = \frac{e^{2x}}{e^x \cdot e^x - e^{-x} \cdot e^{-x} - e^{-x} \cdot e^x + e^{-x} \cdot e^{-x}} = \frac{e^{2x} - 2 + e^{-2x}}{4} = \frac{e^{2x} - 2 + e^{-2x}}{4}.
\]
Putting this together gives
\[ (\cosh x)^2 - (\sinh x)^2 = \frac{e^{2x} + 2 + e^{-2x} - e^{2x} - 2 + e^{-2x}}{4} \]
\[ = \frac{e^{2x} + 2 + e^{-2x} - (e^{2x} - 2 + e^{-2x})}{4} \]
\[ = \frac{4}{4} = 1. \]

Solutions for Chapter 10 Review

EXERCISES

1. The account is increasing in a linear fashion.
2. The machine’s value is depreciating in an exponential fashion.
3. The substance is decaying in an exponential fashion.
4. The quantity of water in the trough is increasing in a linear fashion.
5. The quantity of drug in the body is decaying in an exponential fashion.
6. Here, \( a = 6000 \), \( b = 1.052 \), and \( r = b - 1 = 0.052 = 5.2\% \) per year.
7. Here, \( a = 500 \), \( b = 1.232 \), and \( r = b - 1 = 0.232 = 23.2\% \) per year.
8. Here, \( a = 14,000 \), \( b = 1.0088 \), \( r = b - 1 = 0.0088 = 0.88\% \) per year.
9. Here, \( a = 900 \), \( b = 0.989 \), \( r = b - 1 = -0.011 = -1.1\% \). This investment is initially worth $900 and (because \( r \) is negative) it decreases by 1.1% per year.
10. Since the sunflower is growing at constant percent rate, we use an exponential expression. At 5% per day, the growth factor is \( 1 + 0.05 = 1.05 \). Thus our expression is \( h = a(1.05)^t \). To find \( a \), use our initial condition. The height is 6 inches when \( t = 0 \); thus \( 6 = a(1.05)^0 = a(1) = a \). Therefore \( a = 6 \), and \( h = 6(1.05)^t \).
11. The metal loses 1% of its mass each hour, so its growth factor is \( 1 - 0.01 = 0.99 \), which gives \( w = a(0.99)^t \). Since the metal starts with a mass of 10 grams, we know that when \( t = 0 \), we have \( w = 10 \). Thus \( 10 = a(0.99)^0 = a(1) = a \), giving the expression \( w = 10(0.99)^t \).
12. Yes, \( a = 2 \) and \( b = 5 \).
13. Yes. We can rewrite this as
\[ 10^{-1} = 1 \left( 10^{-1} \right)^t \]
\[ = 1 \cdot 0.1^t, \]
so \( a = 1 \) and \( b = 0.1 \).
14. No. Here, the base involves a variable, and the exponent is constant, so this expression can’t be put in the form \( ab^t \).
15. Yes. We can rewrite this as
\[ 10 \cdot 2^{3+2t} = 10 \cdot 2^3 \cdot 2^{2t} \]
\[ = 80 \cdot 2^t \]
\[ = 80 \cdot 4^t, \]
so \( a = 80 \) and \( b = 4 \).
16. Yes. We can rewrite this as

\[
3^{-2t} \cdot 2^t \cdot 2^t = (3^{-2})^t \cdot 2^t = (\frac{1}{9})^t \cdot 2^t = (\frac{2}{9})^t,
\]

so \(a = 1\) and \(b = 2/9\).

17. Yes. We can rewrite this as

\[
\frac{3}{5}^{2x+1} = \frac{3}{5} \cdot \frac{1}{5} = \frac{3}{25} \cdot \frac{1}{5} = \frac{3}{25} \left(\frac{1}{5}\right)^x,
\]

so \(a = 3/25\) and \(b = 1/5\).

18. We have \(a = 1700, b = 1.117, r = b - 1 = 0.117 = 11.7\%\).

19. We have \(a = 1250, b = 0.923, r = b - 1 = -0.077 = -7.7\%\).

20. We have \(a = 120, b = 3.2, r = b - 1 = 2.2 = 220\%\).

21. We have \(a = 80, b = 0.113, r = b - 1 = -0.887 = -88.7\%\).

22. Exponential, since \(\frac{-21}{2^x} = -21 \left(\frac{1}{2}\right)^x\), which is an exponential expression with constant \(-21\) and base \(1/2\).

23. Exponential, since \(\frac{2^x}{3^x} = \left(\frac{2}{3}\right)^x\), which is an exponential expression of \(x\), with constant \(1\) and base \(2/3\).

24. Power, since \(\frac{x^2}{x^3} = \frac{1}{x} = x^{-1}\), which is a power expression with constant \(1\) and exponent \(-1\).

25. Growth of 60\% corresponds to a growth rate of 0.6. The growth factor is thus \(1 + 0.6 = 1.6\).

26. Shrinkage of 18\% corresponds to a growth rate of \(-0.18\). The growth factor is thus \(1 - 0.18 = 0.82\).

27. Growth of 100\% corresponds to a growth rate of 1. The growth factor is thus \(1 + 1 = 2\).

28. Shrinkage 99\% corresponds to a growth rate of \(-0.99\). The growth factor is thus \(1 - 0.99 = 0.01\).

29. The growth factor is equal to the growth rate + 1; so the growth rate = growth factor - 1. Therefore the growth rate is \(1.095 - 1 = 0.095 = 9.5\%\).

30. The growth factor is equal to the growth rate + 1; so the growth rate = growth factor - 1. Therefore the growth rate is \(0.91 - 1 = -0.09 = -9\%\).

31. The growth factor is equal to the growth rate + 1; so the growth rate = growth factor - 1. Therefore the growth rate is \(2.16 - 1 = 1.16 = 116\%\).

32. The growth factor is equal to the growth rate + 1; so the growth rate = growth factor - 1. Therefore the growth rate is \(0.95 - 1 = -0.05 = -5\%\).

33. Assume the investment begins at \(V_0\) and doubles to \(2V_0\) after 8 years:

\[
V_0 b^8 = 2V_0 \quad b^8 = 2 \quad b = 2^{1/8} = 1.0905,
\]

so \(r = 0.0905 = 9.05\%\).
34. Assume that initially the amount of substance is $Q_0$ and that it halves to $0.5Q_0$ after 11 days:

$$Q_0b^{11} = 0.5Q_0$$
$$b^{11} = 0.5$$
$$b = 0.5^{1/11}$$

so $r = -0.0611 = -6.11\%$ per day.

35. Letting $N_0$ stand for the initial blood count, after 3 days we have

$$N_0b^3 = 0.5N_0$$
$$b^3 = 0.5$$
$$b = 0.5^{1/3}$$

so $r = -0.2063 = -20.63\%$ per day.

36. If $V_0$ is the initial value, after 7 years we have

$$V_0b^7 = 3V_0$$
$$b^7 = 3$$
$$b = 3^{1/7}$$

so $r = 0.1699 = 16.99\%$ per year.

37. Let $a$ represent the initial amount of the radioactive substance. We know that after 25 years, half of the material is left; therefore $a/2 = ab^{25}$. Dividing both sides by $a$, we have $1/2 = b^{25}$. By taking the $1/25^{th}$ power of both sides, we have $(1/2)^{1/25} = (b^{25})^{1/25}$, so $b = (1/2)^{1/25} = 0.9727$. This is the growth factor; the yearly growth rate is then $0.9727 - 1 = -0.0273$, which is a decay rate of 2.73%. Thus the radioactive material loses about 2.73% of its mass per year.

38. We have $B \cdot 2^n$.

39. We have $V(1.04)^n$.

40. Over twelve years, the investment increases by a factor of $k$ three times, or once every four years for twelve years. Its value is therefore given by $V_0 \cdot k^3$.

41. Over twenty years, the investment increases by a factor of $k$ a total of $20/h$ times, or once every $h$ years for twenty years. Its value is therefore given by $V_0 \cdot k^{20/h}$.

42. Over $N$ years, the investment increases by a factor of $k$ a total of $N/h$ times, or once every $h$ years for $N$ years. Its value is therefore given by $V_0 \cdot k^{N/h}$.

43. We have $P_0r^4s^7$.

44. We can use any two of the points to find the equation. We use $(0, 200)$ and $(1, 194)$. We first substitute $(0, 200)$ into the equation $y = ab^x$ to get

$$200 = ab^0$$
$$200 = a \cdot 1$$
$$200 = a.$$  

Using the fact that $a = 200$, we substitute $(1, 194)$ into $y = 200 \cdot b^x$ to get

$$194 = 200 \cdot b^1$$
$$194 = 200 \cdot b$$

$$194/200 = b$$
$$97/100 = 0.97 = b.$$
Thus, our equation is \( y = 200(0.97)^x \).

45. We can use any two of the points to find the equation. We use \((0, 1)\) and \((2, 2)\). We first substitute \((0, 1)\) into the equation \( y = ab^x \) to get

\[
1 = ab^0 \\
1 = a \\
1 = a.
\]

Using the fact that \(a = 1\), we substitute \((2, 2)\) into \( y = 1 \cdot b^x \) to get

\[
2 = 1 \cdot b^2 \\
2 = b^2 \\
\pm \sqrt{2} = b.
\]

We cannot use \(-\sqrt{2}\) however, as we limit the values of the base to positive numbers. Thus, our equation is \( y = 1 \cdot \sqrt{2}^x \) or just \( y = \sqrt{2}^x \). Using a calculator, we have \( y = (1.414)^x \).

46. We can use any two of the points to find the equation. We use \((0, 10)\) and \((3, 9)\). We first substitute \((0, 10)\) into the equation \( y = ab^x \) to get

\[
10 = ab^0 \\
10 = a \cdot 1 \\
10 = a.
\]

Using the fact that \(a = 10\), we substitute \((3, 9)\) into \( y = 10 \cdot b^x \) to get

\[
9 = 10 \cdot b^3 \\
9/10 = b^3 \\
\sqrt[3]{0.9} = b.
\]

Thus, our equation is \( y = 10 \sqrt[3]{0.9}^x \) or just \( y = 10(0.965)^x \).

47. We have \( Q = ab^t \), and we know that this equation is satisfied by \( t = 3, Q = 4 \), so

\[
4 = ab^3.
\]

The equation is also satisfied by \( t = 6, Q = 10 \), so

\[
10 = ab^6.
\]

Dividing the two equations gives

\[
\frac{10}{4} = \frac{ab^6}{ab^3} \\
\frac{5}{2} = b^3.
\]

Solving for \( b \) gives

\[
b = \left(\frac{5}{2}\right)^{1/3} = 1.357.
\]

We now can use either point to solve for \( a \). Using \((3, 4)\), we have

\[
4 = a \left(\left(\frac{5}{2}\right)^{1/3}\right)^3 \\
4 = a \left(\frac{5}{2}\right) \\
a = \frac{8}{5}.
\]
Thus, putting it in the form $Q = ab^t$, we have

$$Q = \frac{8}{5} \left( \frac{5}{2} \right)^{t/3}.$$  

48. We have $Q = ab^t$, and we know that this equation is satisfied by $t = 2, Q = 6$, so

$$6 = ab^2.$$  

The equation is also satisfied by $t = 7, Q = 1$, so

$$1 = ab^7.$$  

Dividing the two equations gives

$$\frac{1}{6} = \frac{ab^7}{ab^2} = \frac{b^5}{1}. $$

Solving for $b$ gives

$$b = \left( \frac{1}{6} \right)^{1/5} = 0.699.$$  

We now can use either point to solve for $a$. Using $(2, 6)$, we have

$$6 = a \left( \frac{1}{6} \right)^{2/5}.$$  

$$6 = a \left( \frac{1}{6} \right)^{2/5} = \frac{6}{(1/6)^{2/5}} = 12.286.$$  

Thus, putting it in the form $Q = ab^t$, we have

$$Q = \frac{6}{(1/6)^{2/5}} \left( \frac{1}{6} \right)^{1/5}.$$  

49. We have $Q = ab^t$, and we know that this equation is satisfied by $t = -6, Q = 2$, so

$$2 = ab^{-6}.$$  

The equation is also satisfied by $t = 3, Q = 6$, so

$$6 = ab^3.$$  

Dividing the two equations gives

$$\frac{6}{2} = \frac{ab^3}{ab^{-6}} = \frac{b^9}{3}.$$  

Solving for $b$ gives

$$b = 3^{1/9} = 1.130.$$  

We now can use either point to solve for $a$. Using $(3, 6)$, we have

$$6 = a \left( 3^{1/9} \right)^3.$$  

$$6 = a 3^{1/3} = a 3^{1/3}.$$  

$$a = \frac{6}{3^{1/3}} = 4.160.$$  

Thus, putting it in the form $Q = ab^t$, we have

$$Q = \frac{6}{3^{1/3}} 3^{1/9}.$$
50. We have \( Q = ab^t \), and we know that this equation is satisfied by \( t = 2 - 5, Q = 8 \), so
\[
8 = ab^{-5}.
\]
The equation is also satisfied by \( t = -2, Q = 1 \), so
\[
1 = ab^{-2}.
\]
Dividing the two equations gives
\[
\frac{1}{8} = \frac{ab^{-2}}{ab^{-5}} = b^3.
\]
Solving for \( b \) gives
\[
b = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2} = 0.5.
\]
We now can use either point to solve for \( a \). Using \((-2, 1)\), we have
\[
1 = a \left(\frac{1}{2}\right)^{-2} = a \left(\frac{1}{2}^{-2}\right) = a (4)
\]
\[
a = \frac{1}{4} = 0.25.
\]
Thus, putting it in the form \( Q = ab^t \), we have
\[
Q = \frac{1}{4} \left(\frac{1}{2}\right)^t.
\]

51. We have \( Q = ab^t \), and we know that this equation is satisfied by \( t = 0 \) and \( Q = 2 \), so
\[
2 = a \cdot b^0 = 2.
\]
The equation is also satisfied by \( t = 3, Q = 7 \), so
\[
7 = ab^3.
\]
Substituting the value of \( a \) gives
\[
7 = 2b^3.
\]
Solving for \( b \) gives
\[
b = \left(\frac{7}{2}\right)^{1/3} = 1.518.
\]
Thus
\[
Q = 2(1.518)^t.
\]

52. We have \( Q = ab^t \), and we know that this equation is satisfied by \( t = 1, Q = 7 \), so
\[
7 = ab^1 = ab.
\]
The equation is also satisfied by \( t = 5, Q = 9 \), so
\[
9 = ab^5.
\]
Dividing the two equations gives
\[
\frac{9}{t} = \frac{ab^5}{ab} = \frac{9}{t} = b^4.
\]
Solving for \(b\) gives
\[
b = \left(\frac{9}{7}\right)^{1/4} = 1.065.
\]
To find \(a\), use the equation \(7 = ab\), with \(b = (9/7)^{1/4}\), so
\[
7 = a \left(\frac{9}{7}\right)^{1/4}
\]
\[
a = \frac{7}{(9/7)^{1/4}} = 6.574.
\]
Thus \(Q = 6.574(1.065)^t\).

53. We have \(Q = ab^t\), and we know that this equation is satisfied by \(t = 3, Q = 6.2\), so
\[
6.2 = ab^3.
\]
The equation is also satisfied by \(t = 6, Q = 5.1\), so
\[
5.1 = ab^6.
\]
Dividing the two equations gives
\[
\frac{5.1}{6.2} = \frac{ab^6}{ab^3} = \frac{5.1}{6.2} = b^3.
\]
Solving for \(b\) gives
\[
b = \left(\frac{5.1}{6.2}\right)^{1/3} = 0.937.
\]
To find \(a\), use the equation \(6.2 = ab^3\), with \(b = (5.1/6.2)^{1/3}\), so
\[
6.2 = a \left(\frac{5.1}{6.2}\right)^{1/3} = a \left(\frac{5.1}{6.2}\right)
\]
\[
a = \frac{6.2}{(5.1/6.2)} = 7.537.
\]
Thus
\[
Q = 7.537(0.937)^t.
\]

54. We have \(Q = ab^t\), and we know that this equation is satisfied by \(t = 2.5, Q = 3.7\), so
\[
3.7 = ab^{2.5}.
\]
The equation is also satisfied by \(t = 5.1, Q = 9.3\), so
\[
9.3 = ab^{5.1}.
\]
Dividing the two equations gives
\[
\begin{align*}
\frac{9.3}{3.7} &= \frac{ab^{5.1}}{ab^{2.5}} \\
\frac{9.3}{3.7} &= b^{5.1-2.5} \\
\frac{9.3}{3.7} &= b^{2.6}.
\end{align*}
\]
Solving for \( b \) gives
\[
b = \left(\frac{9.3}{3.7}\right)^{1/2.6} = 1.425.
\]
To find \( a \), use the equation \( 3.7 = ab^{2.5} \), with \( b = \left(\frac{9.3}{3.7}\right)^{1/2.6} \), so
\[
3.7 = a \left(\frac{9.3}{3.7}\right)^{1/2.6}^{2.5} = a \left(\frac{3.7}{(9.3/3.7)^{2.5/2.6}}\right) = 1.525.
\]
Thus
\[
Q = 1.525(1.425)^t.
\]

55. Since we can put the expression in the form \( at^2 + bt + c \), where \( a = 2, b = 0, c = 7 \), it is quadratic in \( t \).
56. Since we can put the expression in the form \( at^2 + bt + c \), where \( a = 7, b = 8, c = 6m \), it is quadratic in \( t \).
57. This expression is exponential in form.
58. This expression is linear in \( t \), because \( 8 - 4w \) is constant in \( t \).
59. This expression is linear in \( t \) because \( 5h^n \) is constant in \( t \).
60. This expression is exponential in \( t \).

**PROBLEMS**

61. The quantity of caffeine decreases by a factor of \( 1 - 0.17 = 0.83 \) each hour. Since the initial quantity is 90 mg, after \( t \) hours
\[
\text{Quantity of caffeine} = 90(0.83)^t \text{ mg}.
\]

62. The population increases by a factor of 1.01 each year, so
\[
\text{Population} = 281(1.01)^t \text{ million}.
\]

63. Prices increase by a factor of 1.022 each year, so
\[
\text{Price} = 30(1.022)^t \text{ dollars}.
\]

64. The $1000 investment increases by a factor of 1.05 per year, so after \( t \) years, it is worth \( 1000(1.05)^t \).
   The $800 investment increases by a factor of 1.04 per year, so after \( t \) years, it is worth \( 800(1.04)^t \). Thus
\[
\text{Difference in investments} = 1000(1.05)^t - 800(1.04)^t.
\]
65. The first investment increases by a factor of \(1.03\) per year, so after \(t\) years, it is worth \(1500(1.03)^t\).

The second investment increases by a factor of \(1.02\) per year, so after \(t\) years, it is worth \(1500(1.02)^t\). Thus

\[
\text{Difference of investments} = 1500(1.03)^t - 1500(1.02)^t.
\]

66. The expression is increasing by seven percent for each unit increase in \(x\).

67. The expression is decreasing by four percent for each unit increase in \(x\).

68. The expression is increasing. The percent change per unit increase in \(x\) is \(27\%\). This is because, when \(x\) increases by 1, the quantity \(2x\) increases by 2, and \(1.13^2 = 1.2769\).

69. The expression is decreasing. We note first that \(1.05/2 = 0.525\). Thus, for each unit increase in \(x\), the expression decreases by \(47.5\%\).

70. We note that this expression is equivalent to \((5/3)(0.97)^x\). Thus, the expression is decreasing by \(3\%\) for each one unit increase in \(x\).

71. We have \(f(3) = 12\) and \(f(20) = 80\). Using the ratio method, we have

\[
\frac{ab^{20}}{ab^3} = \frac{f(20)}{f(3)}
\]

\[
b^{17} = \frac{80}{12}
\]

\[
b = \left(\frac{80}{12}\right)^{1/17} = 1.11806.
\]

Now we can solve for \(a\):

\[
a(1.11806)^3 = 12
\]

\[
a = \frac{12}{(1.11806)^3} = 8.5859.
\]

so \(f(t) = 8.5859(1.11806)^t\).

72. We have \(g(6) = 3000\) and \(g(14) = 7000\). Using the ratio method, we have

\[
\frac{ab^{14}}{ab^6} = \frac{g(14)}{g(6)}
\]

\[
b^8 = \frac{7000}{3000}
\]

\[
b = \left(\frac{7000}{3000}\right)^{1/8} = 1.1117.
\]

Now we can solve for \(a\):

\[
a \left(\frac{7000}{3000}\right)^{1/8} = 3000
\]

\[
a = \frac{3000}{\left(\frac{7000}{3000}\right)^{1/8}} = 1589.05.
\]

so \(g(t) = 1589.05(1.1117)^t\) dollars.

73. We have \(v(-4) = 8\) and \(v(20) = 40\). Using the ratio method, we have

\[
\frac{ab^{20}}{ab^{-4}} = \frac{v(20)}{v(-4)}
\]

\[
b^{24} = \frac{40}{8}
\]

\[
b = \left(\frac{40}{8}\right)^{1/24} = 1.06936.
\]
Now we can solve for $a$:

$$a(1.06936)^{-4} = 8$$

$$a = \frac{8}{(1.06936)^{-4}} = 10.4613.$$ 

so $v(x) = 10.4613(1.06936)^x$.

74. We have $w(30) = 40$ and $w(80) = 30$. Using the ratio method, we have

$$\frac{ab^{80}}{ab^{30}} = \frac{w(80)}{w(30)}$$

$$b^{50} = \frac{30}{40}$$

$$b = \left( \frac{30}{40} \right)^{1/50} = 0.994263.$$ 

Now we can solve for $a$:

$$a(0.994263)^{30} = 40$$

$$a = \frac{40}{(0.994263)^{30}} = 47.536.$$ 

so $w(t) = 47.536(0.994263)^t$.

75. We know the initial value is $a = f(0) = 2000$. We can use the value in year $t = 5$ to solve for the base $b$:

$$2000b^5 = 5000$$

$$b^5 = 2.5$$

$$b = 2.5^{1/5}$$

$$= 1.201,$$

so $f(t) = 2000(1.201)^t$.

76. Putting the given information together, we have

$$g(8) = 80$$

$$g(11) = 60.$$ 

Since $g$ is exponential, we know that $g(t) = ab^t$. Taking ratios gives

$$\frac{g(11)}{g(8)} = \frac{60}{80}$$

$$\frac{ab^{11}}{ab^8} = \frac{3}{4}$$

$$b^3 = 0.75$$

$$b = 0.75^{1/3}$$

$$= 0.90856.$$ 

Solving for $a$, we have

$$g(8) = 80$$

$$a(0.90856)^8 = 80$$

$$a = \frac{80}{(0.90856)^8} = 172.290,$$

so $g(t) = 172.290(0.90856)^t$. 
77. Since \( P = 7 \) when \( t = 0 \), we want \( P = 14 \) when \( t = 5 \). Substituting \( t = 5 \) into \( P = 7 \cdot 2^{2t} \) gives
\[
P = 7 \cdot 2^{2(5)} = 7 \cdot 2^{25} = 234,881,024.
\]
Thus, the population is multiplied by \( 2^{25} \) in 5 years, rather than doubled. The correct formula is
\[
P = 7 \cdot 2^{t/5}.
\]

78. Since 10% is removed each minute, 90% remains each minute. Thus, the formula should be should be \( Q = Q_0 (0.9)^t \).

79. Let \( Q = Q_0 a^t \), where \( t \) is in years. Then \( Q = 2Q_0 \) when \( t = T \), so
\[
2Q_0 = Q_0 a^T
\]
\[
a^T = 2
\]
\[
a = 2^{1/T}.
\]

80. The population grows by a factor of 1.01 each year, so \( t \) years after 200, the population is \( 282(1.01)^t \). Thus, the equation to be solved is
\[
282(1.01)^t = 300.
\]
Instead of 1.01, the given equation has 0.01 as the growth factor.

81. The population grows by a factor of 1.01 each year, so in 2020, the population will be given by
\[
P = 282(1.01)^{20} \quad \text{or} \quad P - 282(1.01)^{20} = 0.
\]
Instead of \( 282(1.01)^{20} \), the given equation has \( (282 \cdot 1.01)^{20} \), which is incorrect because the 282 should not be taken to the 20th power.

82. The population has doubled when it reaches \( 2 \cdot 282 = 564 \) million, so we solve the equation \( 282(1.01)^t = 564 \) or \( (1.01)^t = 2 \).

The solution to the given equation is the time for the population to reach 2 million, which is in the past.

83. In 30 years, the first investment will double 3 consecutive times, increasing by a factor of 8. The second investment will triple 2 consecutive times, increasing by a factor of 9. Since the second investment grows by more over the same 30-year time period, it grows faster.

84. In 24 years, the first population is halved 2 consecutive times, and so decreases to \( 1/4 \) of its original size. The second investment loses 1/3 of its members every 8 years, and so it decreases by 3 consecutive factors of 2/3 over 24 years. Since \( (2/3)(2/3)(2/3) = 8/27 \), we see that the second population drops to \( 8/27 \) of its original size. Since \( 8/27 > 1/4 \), we see that the first population is disappearing faster.

85. (a) Populations (I), (IV), (V) and (VI) all increase as \( t \) increases.
(b) Populations (II) and (III) decrease as \( t \) increases.
(c) Populations (V) and (VI) grow linearly with time.

86. Since 0.2 is smaller than 1, equation (a) has a negative solution.
Equation (b) can be rewritten as
\[
3 \cdot \frac{1}{4^x} = 1
\]
\[
4^x = 3,
\]
so equation (b) has a positive solution. Since 3 is smaller than 4, the solution is also smaller than 1.
Equation (c) can be rewritten as
\[
\frac{49}{7} = 5^z
\]
\[
7 = 5^z,
\]
so equation (c) has a positive solution. Since 7 is larger than 5, the solution is larger than 1.

Thus, the ordering is
\[
(a) < (d) < (b) < (e) < (c).
\]
87. (a) For the doubling time, we solve the equation \( Q = 2a \), so
\[
2a = ab^t
\]
\[
2 = b^t.
\]
The equation is (III).
(b) For the half-life, we solve the equation \( Q = \frac{1}{2}a \), so
\[
\frac{1}{2}a = ab^t
\]
\[
\frac{1}{2} = b^t
\]
\[
2b^t = 1.
\]
The equation is (II).

88. The equation \( 2 \cdot 5^x = 2070 \) has a positive solution because \( 5^x \) is greater than 1 for \( x > 0 \), so the best-fitting statement is (a).

89. The equation \( 3^x = 0.62 \) has a negative solution because \( 3^x \) is less than 1 for \( x > 0 \), so the best-fitting statement is (b).

90. The equation \( 7^x = -3 \) has no solution because \( 7^x \) is greater than 0 for all \( x \), and so the best-fitting statement is (e).

91. The equation \( 6 + 4.6^x = 2 \) has no solution because \( 4.6^x \) is greater than 0 for all \( x \), and so the best-fitting statement is (e).

92. The equation \( 17 \cdot 1.8^x = 8 \) has a negative solution because \( 1.8^x \) is less than 1 for \( x < 0 \) and so the best-fitting statement is (b).

93. The equation \( 24 \cdot 0.31^x = 85 \) has a negative solution because \( 0.31^x \) is greater than 1 for \( x < 0 \) and so the best-fitting statement is (d).

94. The equation \( 0.07 \cdot 0.02^x = 0.13 \) has a negative solution because \( 0.02^x \) is greater than 0 for \( x < 0 \) and so the best-fitting statement is (d).

95. The equation \( 240 \cdot 0.55^x = 170 \) has a positive solution because \( 0.55^x \) is less than 1 for \( x > 0 \) and so the best-fitting statement is (c).

96. The equation \( \frac{8}{2^x} = 9 \) has a negative solution because \( 2^x \) is less than 1 for \( x < 0 \) and so the best-fitting statement is (b).

97. The equation \( \frac{12}{0.52^x} = 40 \) has a positive solution because \( 0.52^x \) is less than 1 for \( x > 0 \) and so the best-fitting statement is (c).

98. (a) (i) If we write the expression in the form \( a + b \cdot 2^{-t} \), we can see that it is always less than \( b \) plus a positive number, so it always less than \( a + b \).

(ii) Since \( t > 0 \) after the container has been taken from the refrigerator, \( 2^{-t} \) is always less than 1. If we write the expression for the temperature in the form \( a + b(1 - 2^{-t}) \) we see that the temperature is always greater than 0, because it is a plus a positive number.

(b) For \( t = 0 \), the expression becomes \( a - b \cdot 2^0 + b = a \), so \( a \) represents the temperature of the ice cream in the freezer. As \( t \) increases, the term \( b \cdot 2^{-t} \) becomes very small, so \( a + b \) is approximately the temperature of the room. Room temperatures are much colder than refrigerators, so we suppose that the freezer is about \( 10^6 \)F and room temperature is about \( 70^6 \)F, then reasonable values are \( a = 10 \) and \( b = 60 \).

99. Consider \( 10^x \), where \( x \) varies. Our growth factor is greater than 1, so as \( x \) gets larger, \( 10^x \) gets larger. So since \( a < b \), \( 10^a \) will be less than \( 10^b \).

100. Consider \( 10^x \), where \( x \) varies. Our growth factor is greater than 1, so as \( x \) gets larger, \( 10^x \) gets larger. It does not particularly matter that \( b < 0 \) and \( a > 0 \), other than that this means \( a > b \). Thus \( a < b \) but \( 10^a > 10^b \).

We can also notice that since \( b < 0 \), the number \( 10^b \) will be less than 1 (but still positive). Since \( a > 0 \), the number \( 10^a \) is greater than 1, while we know that \( 10^b \) is less than one. Again this shows that \( 10^a \) is greater than \( 10^b \).

101. Again, consider \( 10^x \), where \( x \) varies. Our growth factor is greater than 1, so as \( x \) gets larger, \( 10^x \) gets larger. It does not really matter that \( a \) and \( b \) are less than 0; it only matters that \( a < b \). Therefore \( 10^a \) is less than \( 10^b \).

102. Consider \( x^p \), where \( x \) is some number between 0 and 1 and \( p \) is the power, which can vary. Since \( x \) is between 0 and 1, as \( x \) gets larger, \( x^p \) gets smaller. Therefore \( x^p \) is greater than \( x^p \).

103. Consider \( x^p \), where \( x \) is some number greater than 1 and \( p \) is the power, which can vary. Since \( x \) is greater than 1, as \( x \) gets larger, \( x^p \) gets larger. Therefore \( x^p \) is less than \( x^p \).
104. It does not matter whether or not \( a > b \). Negative numbers are positive when raised to an even power, but they are negative when raised to an odd power. Since \( a \) is even, \( x^a \) will be positive; since \( b \) is odd, \( x^b \) will be negative. Therefore \( x^a \) will be greater than \( x^b \).

105. Consider \( 1/10^x \), where \( x \) varies. Since \( 1/10^x = (1/10)^x \), our growth factor is less than 1. Therefore, as \( x \) gets larger, \( 1/10^x \) gets smaller. So since \( a < b \), \( 1/10^a \) will be greater than \( 1/10^b \).

106. We can also notice that since \( b < 0 \), \( 1/10^b \) will be greater than 1. Since \( a > 0 \), \( 1/10^a \) will be less than 1, again showing that \( 1/10^a \) is less than \( 1/10^b \).

107. Consider \( 1/10^x \), where \( x \) varies. Since \( 1/10^x = (1/10)^x \), our growth factor is less than 1. Therefore, as \( x \) gets larger, \( 1/10^x \) gets smaller. So since \( a < b \), \( 1/10^a \) will be greater than \( 1/10^b \).

108. Linear in \( a \) if \( n = 1 \). Not linear in any variable if \( n \neq 1 \). Since \( 2^n a^n = (2a)^n \), this expression is exponential in \( n \) with base 2.

109. If \( n = 1 \) it is linear in \( a, b \), and \( c \). It is not linear or exponential in any variables if \( n \neq 1 \).

110. This expression is linear in \( A \). If \( q = 1 \) it is linear in \( B \) and \( C \), otherwise not linear in \( B \) or \( C \). Since we can write \( AB^{q}C^{q} = A(BC)^{q} \), this expression is exponential in \( q \) with base \( BC \).

111. Linear in \( A \). If \( t = 1/2 \), then linear in \( b \). Since we can write \( Ab^{2t} = A(b^{2})^{t} \), this expression is exponential in \( t \) with base \( b^{2} \).